Foundations 11 - Week 7: May 25 – 29

Anticipated time required: 3 hours

Goals to be completed:

1. Review mean, median and mode

2. Read through the completed note set for mean, standard deviations, and the normal curve

3. Practice Problems

4. Week 7 Assignment

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Please submit completed assignments to Charlie.feht@yesnet.yk.ca either as a scanned and uploaded PDF attachment to email, or as a jpeg image file.

Upcoming next week:

Course Summary Assignment – Graphical Analysis

**FOM 11** **5.1 Exploring Data**

Statistics is a field of mathematics that deals with the collecting and summarizing of data. There are three measures of central tendency:

1. **Mean** - average
   – Computed by adding a set of values and dividing by the number of values.

2. **Median** – center or middle value
   – Computed by ordering the values from least to greatest, then taking the middle value or the average of the two middle values.

3. **Mode** - most frequent.
   – Computed by taking the value that occurs the most often.

**Example 1:** For the set of values: 1, 6, 3, 8, 9, 3, 6, 1, 6 determine the:

a. mean (Average) Add all & divide by the number of terms.

$$\frac{1+6+3+8+9+3+6+1+6}{9} = \frac{43}{9} = \boxed{4.78}$$

b. median (Middle) Order smallest to largest & take middle # (or average of 2 middle #s)

1, 1, 3, 3, 6, 6, 6, 8, 9

$\boxed{\text{Median} = 6}$

c. mode (Most Common)

(If 2 #'s occur the most then there is no mode)

$\boxed{\text{Mode} = 6}$

Upper Case "sigma"

$\sum$ means sum (add all terms together)

To describe data numerically, we often use two numbers:

1. **Mean**: the average

Let $x_1, x_2, x_3, \ldots, x_n$ represent any set of values.

"sigma" (upper case)

Mean: $\bar{x} = \mu = \dfrac{\sum x_i}{n} = \dfrac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$

"mu"

2. **Standard Deviation:** a measure of the extent to which data cluster around the mean.

Let $x_1, x_2, x_3, \ldots, x_n$ represent any set of values.

lower case "sigma"

Standard Deviation: $\sigma = \sqrt{\dfrac{\sum (x_i - \mu)^2}{n}} = \sqrt{\dfrac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}}$

The smaller the standard deviation, the more consistent the results and the closer the data to the mean.

**Example 1:** Calculate the standard deviation from the following sets of values:

a. 7, 8, 9, 10, 11 ($x_1\ x_2\ x_3\ x_4\ x_5$)

$\text{Mean} = \mu = \dfrac{7+8+9+10+11}{5} = 9$

Standard Deviation:

$$\sigma = \sqrt{\frac{(7-9)^2 + (8-9)^2 + (9-9)^2 + (10-9)^2 + (11-9)^2}{5}}$$

$$= \sqrt{\frac{(-2)^2 + (-1)^2 + (0)^2 + (1)^2 + (2)^2}{5}}$$

$$= \sqrt{\frac{4+1+0+1+4}{5}} = \sqrt{\frac{10}{5}} = \sqrt{2} = \boxed{1.41}$$

On Graphing Calc:

STAT → 1: Edit

Clear L1 & enter all values

STAT ▶ CALC

1: 1-var stats

$\bar{x} = 9$ (mean)

$\sigma x = 1.41$ (standard deviation)

b. 7, 9, 11, 13, 15 $\mu = \frac{7+9+11+13+15}{5} = 11$

$$\sigma = \sqrt{\frac{(7-11)^2+(9-11)^2+(11-11)^2+(13-11)^2+(15-11)^2}{5}}$$

$$= \sqrt{\frac{(-4)^2+(-2)^2+(0)^2+(2)^2+(4)^2}{5}}$$

$$= \sqrt{\frac{16+4+0+4+16}{5}}$$

$$= \sqrt{\frac{40}{5}} = \sqrt{8} = \boxed{2.83}$$

$\boxed{\text{STAT}}$ 1: Edit

clear L1 & enter #s

$\boxed{\text{STAT}}$ ▶ CALC

1: 1-Var Stats

$\bar{x} = 11$ (mean)

$\sigma x = 2.83$ (standard deviation)

**Example 2:** Calculate the standard deviation for the following sets of data:

a. *Use Middle Value to Find Mean ($\mu$)

| Midpoint | Daily Commute Time (min) | Number of Employees |
|---|---|---|
| 5 | 0-10 | 4 |
| 15 | 10-20 | 9 |
| 25 | 20-30 | 6 |
| 35 | 30-40 | 4 |
| 45 | 40-50 | 2 |

Total = 25

$$\mu = \frac{5(4)+15(9)+25(6)+35(4)+45(2)}{25}$$

$$= \frac{535}{25} = 21.4$$

# Employes Midpoint

$$\sigma = \sqrt{\frac{4(5-21.4)^2+9(15-21.4)^2+6(25-21.4)^2+4(35-21.4)^2+2(45-21.4)^2}{25}}$$

$$= \sqrt{\frac{4(268.96)+9(40.96)+6(12.96)+4(184.96)+2(556.96)}{25}}$$

$$= \sqrt{\frac{3376}{25}} = \sqrt{135.04}$$

$$\boxed{= 11.62}$$

$\boxed{\text{STAT}}$ 1: Edit

$L_1$ = Midpoint of Intervals

$L_2$ = # in each interval

$\boxed{\text{STAT}}$ ▶ CALC

1: 1 Var STAT $L_1, L_2$

$\boxed{\text{2ND}}$ $\boxed{1}$ & $\boxed{2}$

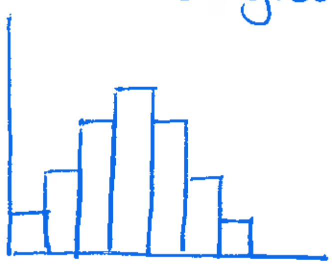
# 5.4 The Normal Distribution

**Normal Distribution:** Data that, when graphed as a histogram or frequency polygon, results in a unimodal symmetric distribution about the mean.

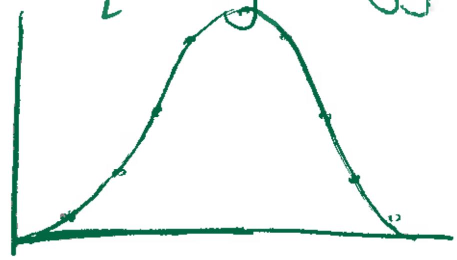
One Mode
(the mean is also the mode)

The **normal curve** is a symmetrical curve that represents the normal distribution. It is also called a **bell curve**.

Histogram



Frequency Polygon



**Properties of a normal distribution:**

- Has a mean $\mu$ and a standard deviation $\sigma$.
- Symmetrical about the mean.
- Almost all the population lies within 3 standard deviations of the mean.
- The horizontal axis is an asymptote. (Never touches horizontal axis)
- The total area under the curve is 1.
- It follows the 68-95-99.7 Rule.

68% within $2\sigma$ of mean

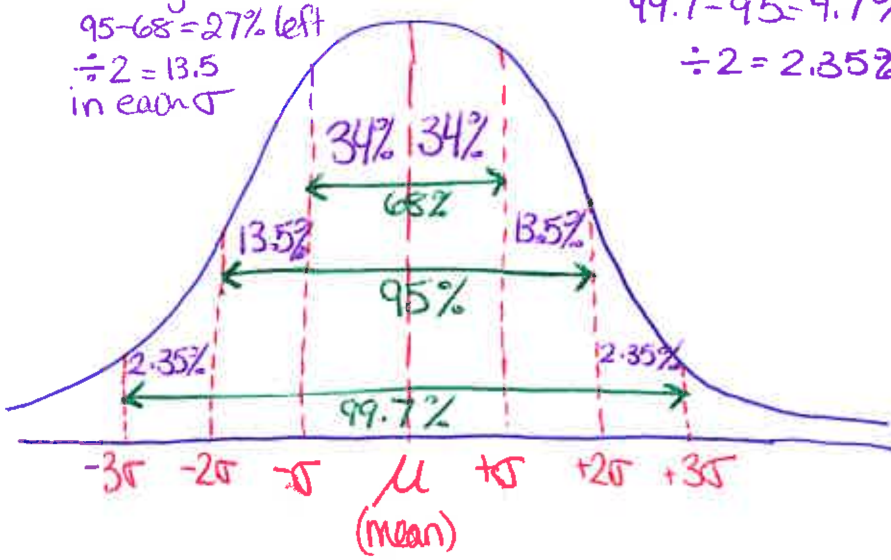
34% on each side of mean

95% within $4\sigma$ of mean

already used 68%
95−68=27% left
÷2 = 13.5
in each $\sigma$

99.7% within $6\sigma$ of mean

already used 95%
99.7−95=4.7% left
÷2 = 2.35% in each $\sigma$



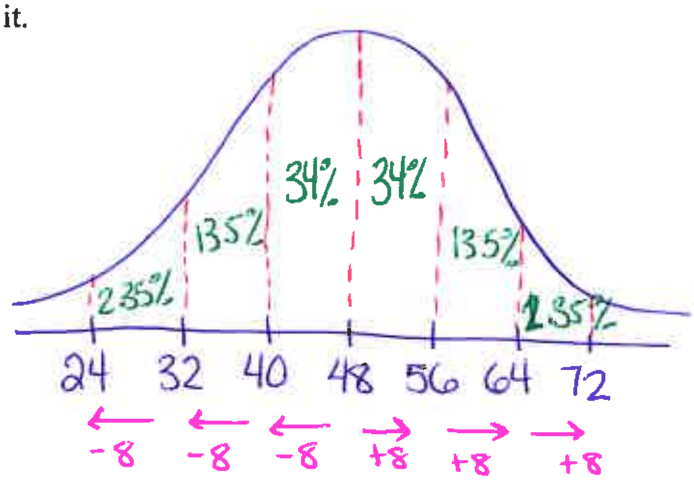
**Example 1:** A company has determined that the lifetime of the car battery it manufacturers is normally distributed with a mean of 48 months and a standard deviation of 8 months.

Symmetrical About $\mu$

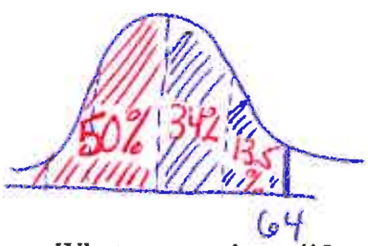
middle

each section

a. Sketch it.



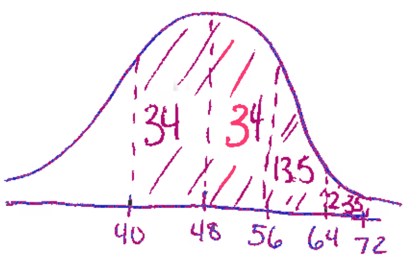

b. What percent have life spans less than or equal to 64 months.




50% + 34% + 13.5%

$\boxed{= 97.5\%}$

c. What percent have life spans between 40 and 72 months?



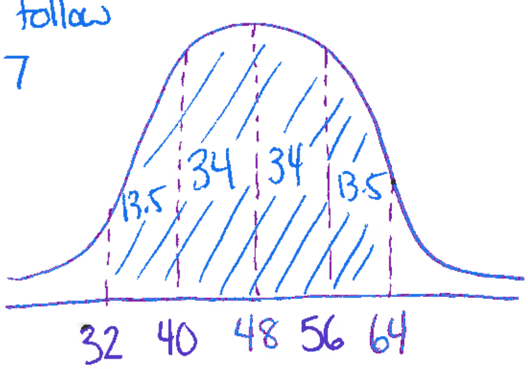

34% + 34% + 13.5% + 2.35%

$\boxed{= 83.85\%}$

d. Between which life spans do 95% of the batteries lie?

Normal Distributions follow 68-95-99.7 Rule

∴ 95% within 2 Standard deviations



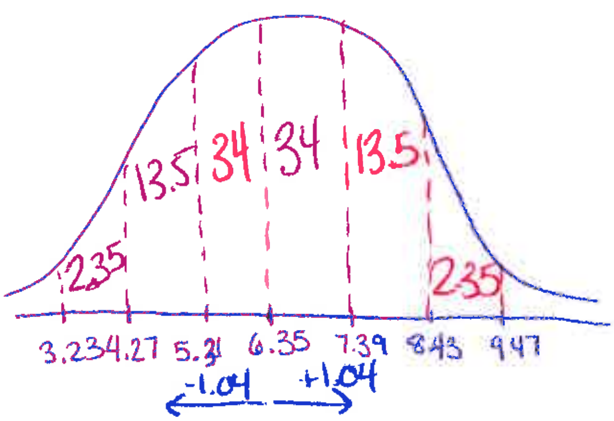

$\boxed{\text{32-64 months}}$

**Example 2:** Two baseball teams flew to the North American Indigenous Games. The members of each team had carry-on luggage for their sports equipment. The masses of the carry-on luggage were normally distributed, with the characteristics shown in the table.

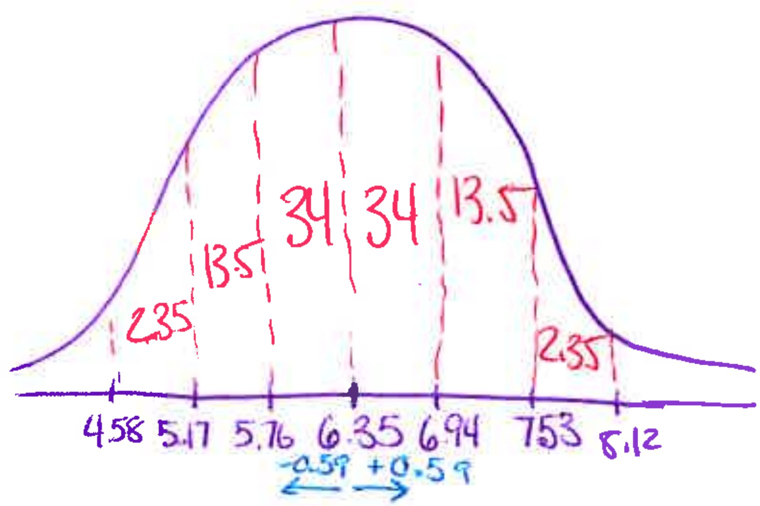
| Team | $\mu$ (kg) | $\sigma$ (kg) |
|---|---|---|
| Men | 6.35 | 1.04 |
| Women | 6.35 | 0.59 |

a. Sketch a graph to show the distribution of the masses of the luggage for each team.

Men

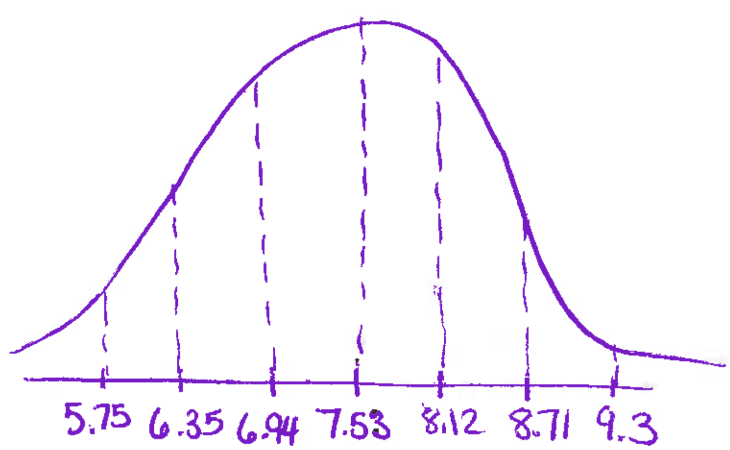



Women




b. The women's team won the championship. Each member received a medal and a souvenir baseball, with a combined mass of 1.18 kg, which they packed in their carry-on luggage. Sketch a graph that shows the distribution of the masses of their carry-on luggage change for the flight home.

$\mu = 6.35 + 1.18$
$= 7.53$
$\sigma = 0.59$



## Practice Problems

1. Use the central tendency (mean, median and mode) to compare the results for two tests given by the same teacher to the same class in the same semester. Did the class perform better on the first or second test? Explain your decision.

| Unit 1 Test Scores (%) | | | Unit 2 Test Scores (%) | | |
|---|---|---|---|---|---|
| 81 | 61 | 68 | 98 | 73 | 73 |
| 71 | 79 | 73 | 95 | 73 | 71 |
| 73 | 74 | 75 | 93 | 73 | 69 |
| 71 | 73 | 79 | 89 | 73 | 64 |
| 64 | 67 | 63 | 87 | 76 | 59 |
| 80 | 58 | 71 | 84 | 79 | 59 |
| 75 | 78 | 73 | 81 | 79 | 57 |

2. Determine the standard deviation of test marks for two different math classes shown. Which class had the more consistent marks over the first five tests? Explain.

| Test | Class A (%) | Class B (%) |
|---|---|---|
| 1 | 94 | 84 |
| 2 | 56 | 77 |
| 3 | 89 | 76 |
| 4 | 67 | 81 |
| 5 | 84 | 74 |

3. A teacher is analyzing the class results from three chemistry tests. Each set of marks is normally distributed.

| Test | Mean ($\mu$) | Standard deviation ($\sigma$) |
|---|---|---|
| 1 | 77 | 3.9 |
| 2 | 83 | 3.9 |
| 3 | 77 | 7.4 |

   a. Sketch the normal curve distribution for test 1 and 2 on one graph. Sketch the normal curve distribution for test 1 and 3 on one graph.
   b. Examine the graphs, how do tests 1 and 2 compare? How do tests 1 and 3 compare?
   c. Determine Oliver's mark on each test given the information shown below.

| Test | Oliver's Mark |
|---|---|
| 1 | $\mu + 2\,\sigma$ |
| 2 | $\mu - 1\,\sigma$ |
| 3 | $\mu + 3\,\sigma$ |

**Formal Assignment**

1. A pear orchard has trees with these heights listed in inches. Determine the standard deviation to 1 decimal place.

110, 83, 104, 95, 88, 80, 115, 106

2. What is meant by the 68 – 95 – 99.7 rule?

3. The ages of members of a seniors curling club are normal distributed, with a mean of 63 years and a standard deviation of 4 years. What percent of curlers is in each of the following age groups?

    a. Between 55 and 63 years
    b. Between 67 and 75 years
    c. Older than 75 years

4. A teacher is analyzing the class results from a computer science test. The marks are normally distributed with a mean of 79.5 and a standard deviation of 3.5. Sketch a normal distribution for the test.