Foundations 11 - Week 4: May 4 – May 8

Anticipated time required: 3 hours

New learning objective: **Solving a quadratic by factoring and the quadratic formula**

Goals to be completed:

1. Become proficient in factoring a quadratic to solve the zeros
2. Learn to apply the quadratic formula when factoring does not yield an answer

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This PDF package contains several notes, examples and practice problems. The only formal portion that you are required to submit is the section at the end of this package. This can be sent to Charlie.feht@yesnet.yk.ca either as a scanned and uploaded PDF attachment to email, or as a jpeg image file. Midterm assignments will be scored and sent back to you as I receive them.

Upcoming next week:

Application of vertex form and quadratics summary assignment

<u>Section 1: Factored form of a quadratic</u>

So far, we have seen quadratic equations represented in *standard* form $y = ax^2 + bx + c$

This form of a quadratic is helpful for us because it can directly tell us where the y intercept is, and the direction of opening (as well as width) of a parabola. The standard form quadratic equation does not, however, tell us what the zeros of the parabola are. For that we require the factored form of a quadratic equation.

The factored form of a quadratic looks like this: $\mathbf{y = a\,(x \pm b)\,(x \pm c)}$

In this case, the x variables represent the roots of the parabola when the equation is set to zero: $\mathbf{a\,(x \pm b)\,(x \pm c) = 0}$

The values a, b, and c are all constants.

Proof:

When we set the value of 'y' to zero, we are solving the equation for the x intercepts (remember, when y=0 we are crossing the x axis). So, consider the following:

In the equation $2(x+1)(x-2) = 0$ we can see that we need the left side terms to equal the right side terms (0). Think to yourself, what values of x will make that true?

Lets look at each one individually:

$(x+1) = 0$ — Left side must equal right side

$x + 1 = 0$

$x + 1 - 1 = 0 - 1$ — Subtract 1 from both sides in order to isolate x

$x = -1$

x is now equal to – 1

$(x-2) = 0$ — Left side must equal right side

$x - 2 = 0$

$x - 2 + 2 = 0 + 2$ — Add 2 to both sides in order to isolate x

$x = +2$

x is now equal to + 2

The zeros are at x = +2 and x = -1

Tip: we can ignore the coefficient of 2 in front of the equation, because if we solve what value of x sets the term to 0, then 2 x 0 = 0 and we have matched the right side!

Relationship:

Whatever the coefficient value is within the brackets, as long as the equation is set to zero, the root will be the opposite sign of that coefficient.

Example:

The roots of the equation $(x + 4)(x - 3) = 0$ are…

$x = -4$ and $+3$

Practice:

1. What are the roots for the following equations:
   a. $y = 2(x + 1)(x + 5)$

   b. $y = -2(x + 1)(x - 3)$

   c. $y = (x - 0.5)(x - 2.5)$

   d. $y = x(x + 4)$

Suppose you are not given an equation in factored form. Rather, you are given an equation in standard form. In order to get from standard form to factored form, we need to factor the quadratic!

Case 1: factoring a quadratic when a = 1:

1. Remove the greatest common factor from the quadratic (if there is one)
2. Split your quadratic into a factored form equation (x __)(x___)
3. Find two numbers that multiply to 'c' and add to 'b'
4. Place those two numbers into your factored form

Ex. 1.

$2x^2 + 6x = 0$

$2x(x + 3) = 0$

Rule #1: 2x can be evenly factored out of both terms

$2x = 0$ $\quad\quad$ $(x + 3) = 0$

$x = 0$ $\quad\quad$ $x = -3$

No Common Factor

Rule 2

Rule 3

Ex 2. $x^2 - x - 6 = 0$

$(x\_\_\_)(x\_\_\_) = 0$

$(x - 3)(x + 2) = 0$

Rule 4

$x = 3, -2$

_-3_ x _+2_ = -6

_-3_ + __+2_ = -1

<u>Case 2: factoring a quadratic when a ≠ 1:</u>

Let's look at the following example: $6x^2 + 5x - 4$

1) **Look for a GCF**: There is no GCF for this trinomial and the only way this method works is if you take it out right away.

2) **Take the coefficient for x^2 (6) and multiply it with the last term (4):**

   $6x^2 + 5x - 4$ $\qquad$ $6 * 4 = 24$

   $x^2 + 5x - 24$

3) **Factor the new trinomial using Case I**:

   $x^2 + 5x - 24$

   $(x + 8)(x - 3)$

4) **Take the coefficient that you multiplied in the beginning (6) and put it back in the parenthesis (only with the x):**

   $(x + 8)(x - 3)$

   $(6x + 8)(6x - 3)$

5) **Find the GCF on each factor (of each set of parenthesis):**

   $(6x + 8) = 2(3x + 4)$

   $(6x - 3) = 3(2x + 1)$

6) **Keep the factor left in parenthesis**:

   $(3x + 4)(2x - 1)$

Solve the zeros:

$(3x + 4) = 0$	$(2x - 1) = 0$
$3x + 4 - 4 = 0 - 4$	$2x - 1 + 1 = 0 + 1$
$3x = -4$	$2x = +1$
$x = -4/3$	$x = ½$

Practice factoring the following standard form quadratics to find the zeros.

1. $x^2 + 8x + 7$
2. $x^2 - 11x + 10$
3. $x^2 - 10x + 24$
4. $x^2 - x - 2$
5. $2x^2 + 12x + 10$
6. $2x^2 + 15x + 7$
7. $3x^2 - 5x - 12$
8. $9x^2 + 11x + 2$
9. $7x^2 - 22x + 3$
10. $18x^2 - 9x - 2$

Section 2: The Quadratic Formula

Although it may look scary, the quadratic formula is an extremely helpful way of solving the zeros of a quadratic when factoring does not produce a nice set of x values. The quadratic formula uses the coefficients a, b, and c from a standard form quadratic to solve the roots of a parabola. It looks like this:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex. Solve each equation:

a. $6x^2 - 3 = 7x$

$6x^2 - 7x - 3 = 0$

A = 6
B= -7
C = -3

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 - (-72)}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

X = (7 + 11)/12
X = 18/12
X = 3/2

X = (7 – 11)/12
X = - 4/12
X = -1/3

b. $12x^2 - 47x + 45 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A = 12
B= -47
C = 45

$$x = \frac{-(-47) \pm \sqrt{(-47)^2 - 4(12)(45)}}{2(12)}$$

$$x = \frac{47 \pm \sqrt{2209 - 2160}}{24}$$

$$x = \frac{47 \pm \sqrt{49}}{24}$$

$$x = \frac{47 \pm 7}{24}$$

X = (47+7)/24
X = 54/24
X = 9/4

X = (47-7)/24
X = 40/24
X = 5/3

Practice:

1. Solve $2x^2 + 8x - 5 = 0$
2. Solve $5x^2 - 10x + 3 = 0$

IMPORTANT NOTE: It is impossible to take the square root of a negative number. If you end up with a negative value underneath the square root sign, that means that there are NO ROOTS TO THE PARABOLA

Assignment

Solve the following standard form quadratics by factoring

1) $a^2 + 3a + 2$	2) $c^2 + 6c + 5$	3) $x^2 – 6x – 7$	4) $r^2 + 12r + 11$
5) $m^2 + 5m + 4$	6) $y^2 + 12y + 35$	7) $x^2 + 11x + 24$	8) $a^2 + 11a + 18$
9) $16 + 17c + c^2$	10) $x^2 + 2x + 1$	11) $z^2 + 10z + 25$	12) $a^2 – 8a + 7$
13) $a^2 – 6a + 5$	14) $x^2 – 5x + 6$	15) $c^2 – 2c – 15$	16) $y^2 – 6y + 8$
17) $15 – 8y + y^2$	18) $x^2 – 13x – 48$	19) $c^2 – 14c + 40$	20) $x^2 – 16x + 48$
21) $3x^2 - 20x – 63$	22) $3x^2 - 20x – 7$	23) $8x^2 + 13x – 6$	24) $4x^2 - 17x – 42$
25) $2x^2 - 9x – 18$	26) $6x^2 + 17x – 14$	27) $3x^2 + 5x – 12$	28) $2x^2 + 9x + 4$

Solve the following standard form quadratics using the quadratic formula

1. $3x^2 + 5x = 9$
2. $1.4x – 3.9x^2 = -2.7$
3. $6x – 3 = 2x^2$
4. $X^2 + 1 = x$