Foundations 11 - Week 3: April 27 – May 1

Anticipated time required: 3 hours

New learning objective: **Graphing quadratics and analyzing their behaviour**

Goals to be completed:

1. Revisit the properties all quadratics demonstrate and determine behaviour of parabolas, including domain and range
2. Practice graphing quadratics with a table of values and technology to solve parabolas

---

This PDF package contains several notes, examples and practice problems. The only formal portion that you are required to submit is the section at the end of this package. This can be sent to Charlie.feht@yesnet.yk.ca either as a scanned and uploaded PDF attachment to email, or as a jpeg image file. Midterm assignments will be scored and sent back to you as I receive them.

Upcoming next week:

Factoring quadratics and using factored form of a quadratic to solve parabola behaviour

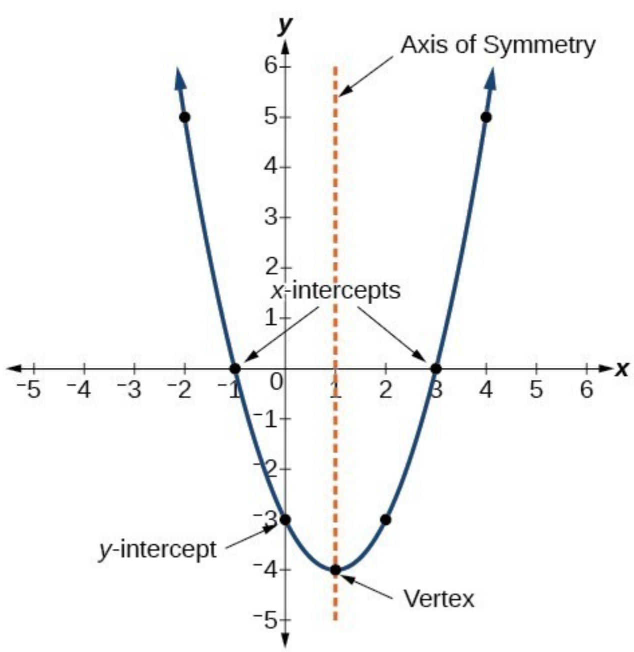
Section 1: Properties of quadratics and parabola behaviour

Recall that all parabolas contain the following properties that describe their "behaviour":

1. **Vertex**: given as a coordinate point, tells you the maximum or minimum point on the graph
2. **Y – intercept**: given as a coordinate point, represents where the graph crosses the y-axis (x will always be 0 at this point)
3. **X – intercept**: given as a coordinate point, represents where the graph crosses the x-axis (y will always be 0 at this point)
4. **Axis of symmetry**: the vertical line on the parabola that cuts the graph perfectly in half. This is always the same as the x coordinate of the vertex.
5. **Domain**: the set of all the valid x values for the graph
6. **Range:** the set of all the valid y values on the graph

Domain: spans all the values that this graph touches on the x axis. Domain deals with all possible input values for x

For this graph, the parabola will eventually touch all of the positive and negative x values. We refer to this by saying X is an element of all real numbers. It is written in mathematical terms as: $x \in \mathbb{R}$



Range: spans all the values that this graph touches on the y axis. Domain deal with vertical position.

For this graph, the parabola will touch all of the y values that are greater than or equal to – 4 because – 4 is the minimum point. Written in mathematical terms we say: $y \geq -4, \in \mathbb{R}$

Y must be greater than or equal to – 4 and is an element of all real numbers

Please watch the two videos below:

1. Review of domain and range (watch up to the 2:45 mark):

   https://www.youtube.com/watch?v=FdK9_Fp76cw

2. Domain and range for a parabola:
   https://www.youtube.com/watch?v=za0QJRZ-yQ4

Example 1. Let's describe the behaviour of the following quadratic functions, and use that information to sketch a graph of what it looks like.

$$Y = -x^2 - 2x + 3$$

First, we must set up a table of values to Determine a few key points on the graph

X Input	Substitutions into $Y = -x^2 - 2x + 3$	Y Output
-3	$Y = -(-3)^2 - 2(-2) + 3$	-2
-2	$Y = -(-2)^2 - 2(-1) + 3$	3
-1	$Y = -(-1)^2 - 2(-1) + 3$	4
0	$Y = -(0)^2 - 2(0) + 3$	3
1	$Y = -(1)^2 - 2(1) + 3$	0
2	$Y = -(2)^2 - 2(2) + 3$	-5
3	$Y = -(3)^2 - 2(3) + 3$	-12

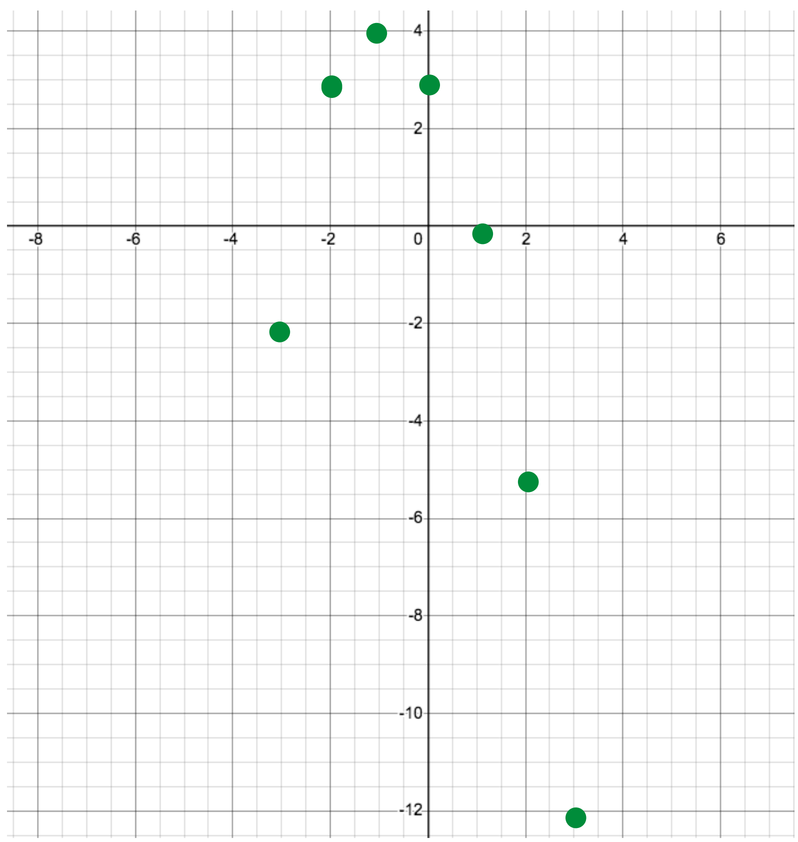
REMEMBER!

What do we already know about this function?

The leading coefficient (the 'a' term) is a negative so this function will open downwards and have its vertex at a maximum point

The c constant is +3 which means that the y intercept is +3

Now That we have coordinate pairs,
We can plot the points on the graph →



HINT:

The axis of symmetry of a function will be Directly between 2 identical y values. Do you see any?

Yes! There are two identical y values in this function, the value of 3 as I have highlighted.

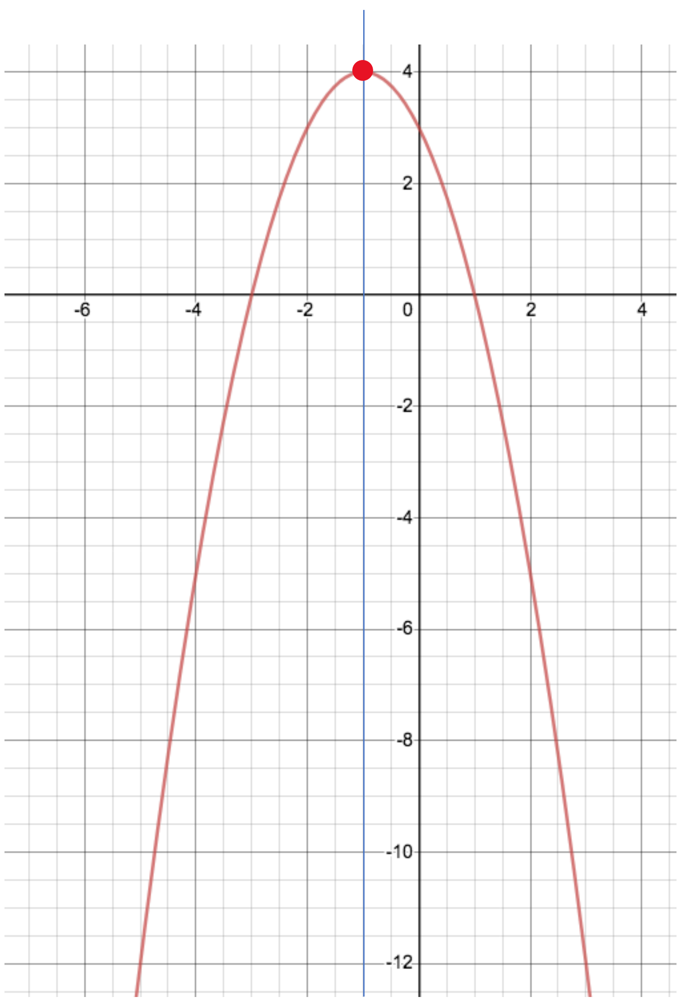
This means that the axis of symmetry occurs directly between the two input values (the two x values) that gave identical outputs. In our case, the x value of – 2 and the x value of 0, both produced a y value of 3. That means that the axis off symmetry occurs directly between these two x points.

What is the point directly between x = -2 and x = 0? Well, (-2 + 0) / 2 = -1 . The axis of symmetry is at -1.

Add the two values, divide by 2.
Like taking an average

Now then, if the axis of symmetry is at -1, and we know that the axis of symmetry is always the x coordinate of the vertex, we can solve our exact vertex point! Simply input the -1 x value into the equation to solve the output y value.

The vertex coordinate is (-1, 4)



With all of this information, we can clearly begin to sketch an accurate representation of our parabola and describe its behaviour.

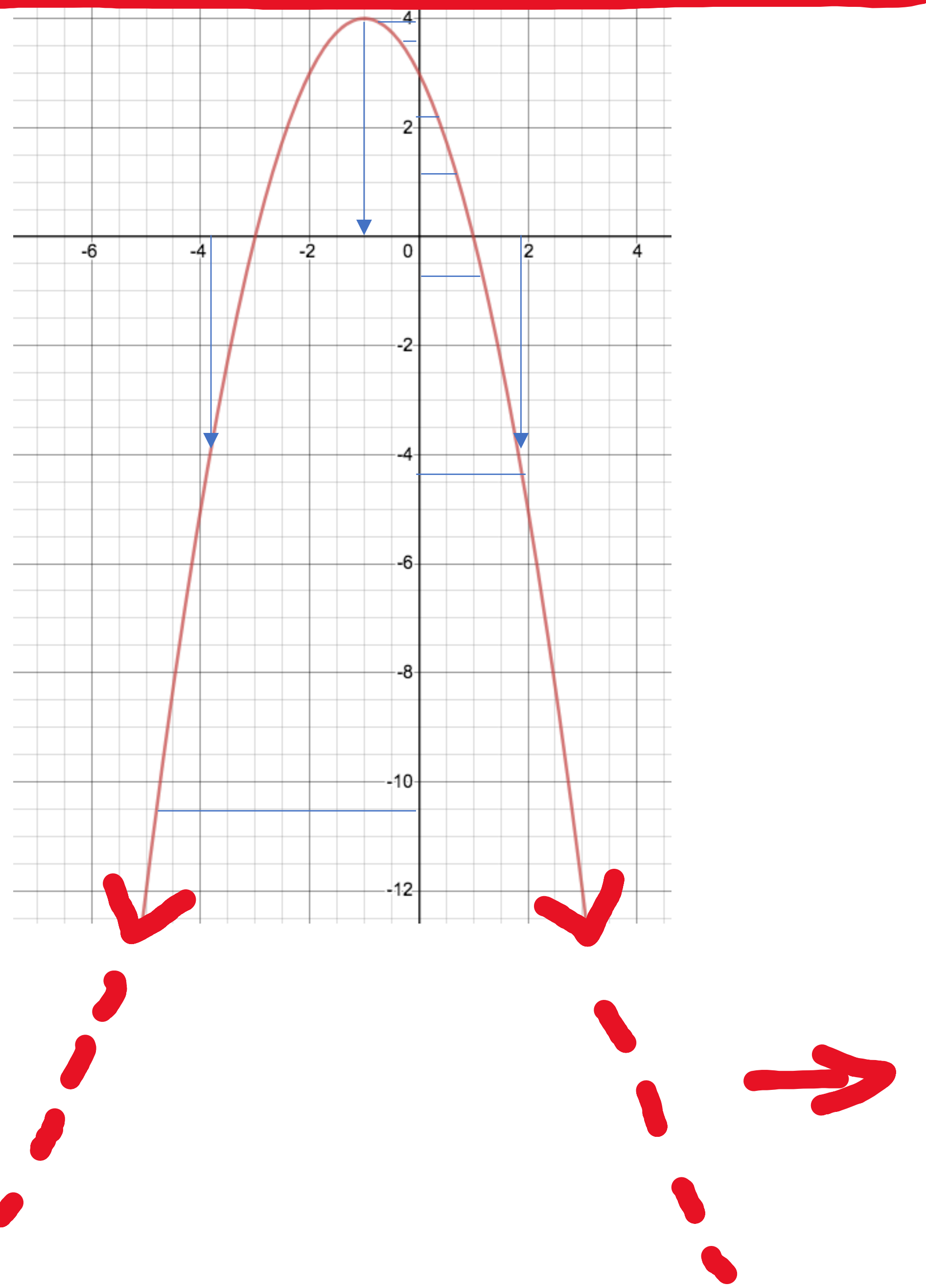
We have identified the vertex, axis of symmetry and also know that it opens down. We can also see the x and y intercepts:

Vertex: (-1, 4)
Y- intercept: 3
X intercept: -3 and 1
Axis of symmetry: x = -1

Domain: X ∈ ℝ
Range: Y ≤ 4 ∈ ℝ

Double check on Domain and Range

The range of this function, aka the possible output y values MUST be less than 4. The function does not exist above the value of 4, but it will continue forever as the function moves towards negative infinity on the y axis



The domain of this function could be ANY input value of x, because the ends of the parabola will continue to grow and expand outwards in both positive and negative x axis directions up to infinity

Practice problem #1:

Sketch a graph of the given function, and describe its properties without the use of technology

$$y = x^2 + x - 2$$

Vertex:
Y-intercept:
X-intercept:
Axis of symmetry:
Domain:
Range:

Practice Problem #2:

Some boaters use red aerial miniflares in an emergency. The path of one brand of flare, when fired at an angle of 70° to the horizon, is modelled by the function

$$h(t) = -9(t-3)^2 + 83$$

Where $h(t)$ is the height in metres and t is the time in seconds since the flare was fired. You may use desmos to assist you in answering the questions below. https://www.desmos.com/calculator

a. Sketch a graph of this function

b. What is the maximum height of the flare?

c. How many seconds will pass before the flare hits the water?

d. If the flare can burn red for 2 seconds, how high is it when it burns out?

Section #2: Solving quadratic functions by graphing

To solve a quadratic function means to find the x-intercepts of the parabola. The x intercepts are where the parabola crosses the x axis, and may be referred to as the zeroes, or as the roots of the equation.

Recap, a **quadratic function** is described by $y = ax^2 + bx + c$

A **quadratic equation** is described as $ax^2 + bx + c = 0$

actual value indicates a real equation, specifically in this case, when y=o or when a line crosses the y axis.

Use the desmos graphing app, to solve the following quadratic equations
https://www.desmos.com/calculator

Tip: when using the desmos app, type in the quadratic function only. Do not put in the equation that includes " = 0". To solve, simply hover your mouse over the x intercepts.

Practice Problems:

1. Solve $3x^2 - 11x - 4 = 0$

2. Find the zeroes for $y = x^2 - x - 20$

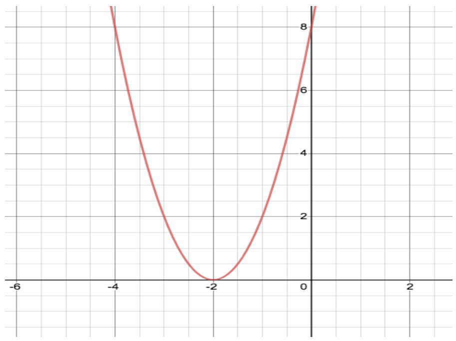
3. What are the roots for $x^2 - 6x + 9 = 0$

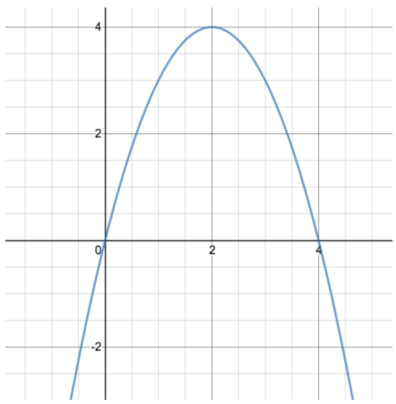
4. The manager at a fashion store has determined that the function $R(x) = 600 - 6x^2$ models the expected weekly revenue, *R*, in dollars, from sweatshirts price changes, where *x* is the change in price, in dollars. What price increase or decrease will result in no revenue?

Here is a great recap on quadratic equations:
https://www.youtube.com/watch?v=UZTvYYoOrmI

As well as how to find the zeros of a quadratic equation:
https://www.youtube.com/watch?v=LFYd_u9vqRc

Week 3 Assignment

1. Describe the Domain, Range, Vertex, Axis of symmetry, x-intercepts, and y intercepts of the following functions (you may use technology):
    a. $y = x^2 - 5x - 6$
    b. $y = -x^2 + 6x - 10$
    c. $y = -x^2 + 8x + 12$
    d. $y = x^2 - 4x + 3$
    e. 

    f. 

2. For each of the following, both points (x, y) are located on the same parabola. Determine the equation of the axis of symmetry for each parabola.
    a. (0, 2) and (6, 2)
    b. (1, -3) and (9, -3)
    c. (-6, 0) and (2, 0)
    d. (-5, -1) and (3, -1)

3. In a football game, a kicker kicks the ball for a field goal modelled by the equation:

$$y - -4.9x^2 + 25x$$

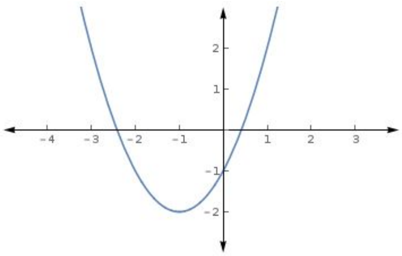
Determine the maximum height of the ball that is kicked, and determine how long the ball is in the air for.

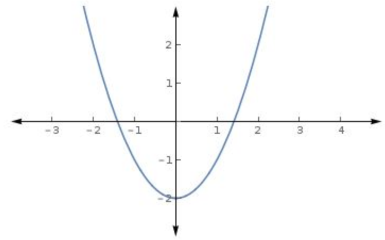
4. Solve each equation by graphing the corresponding function determining the zeros
    a. $3x^2 - 6x - 7 = 0$
    b. $0.5x^2 + 3x - 2 = 0$
    c. $3x^2 + 8x + 7 = 0$
    d. $5p = 3 - 2p^2$

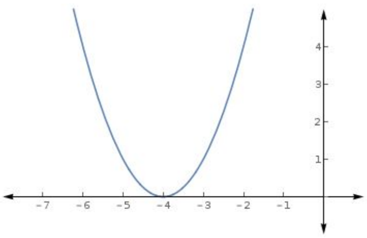
Name: ______________________________ Teacher: ________________

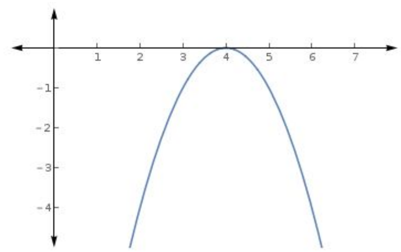
Block: ______

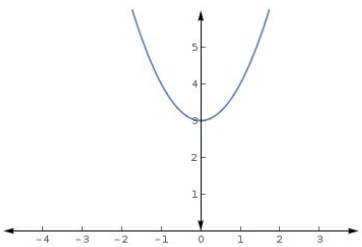
Match the quadratic function with its graph.

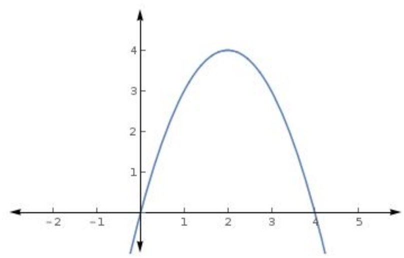

(a)

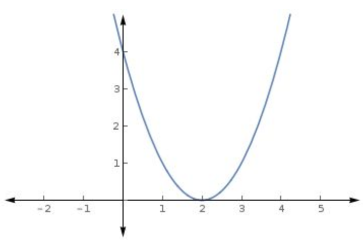

(b)

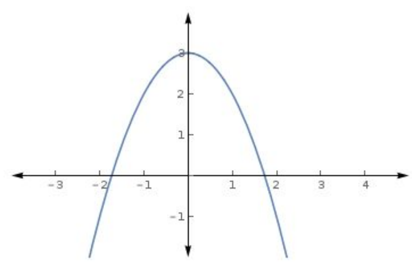

(c)


(d)


(e)


(f)


(g)


(h)

1. $f(x) = (x - 2)^2$

2. $f(x) = (x + 4)^2$

3. $f(x) = x^2 - 2$

4. $f(x) = 3 - x^2$

5. $f(x) = 4 - (x - 2)^2$

6. $f(x) = (x + 1)^2 - 2$

7. $f(x) = x^2 + 3$

8. $f(x) = -(x - 4)^2$