Foundations 11 - Week 2: April 20 – 24

Anticipated time required: 3 hours

New learning objective: **Exploring quadratic relations**

Goals to be completed:

1. Learn what a quadratic function is
2. Explore different parabola properties
3. Graphing parabolas using a table of values and using technology

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This PDF package contains several notes, examples and practice problems. The only formal portion that you are required to submit is the section titled "Graphing Quadratic Functions". This can be sent to Charlie.feht@yesnet.yk.ca either as a scanned and uploaded PDF attachment to email, or as a jpeg image file. Midterm assignments will be scored and sent back to you as I receive them.

Upcoming next week:

Graphing Quadratics and Analyzing their Properties

Section 1: What is a quadratic function?

A *quadratic function* is any type of function that has a degree of 2 (ie. The highest exponent found on any variable is 2).

You have seen functions before in math 10, given by the equations y = mx + b. This is the slope of a *linear function*, where m = line slope, b = y intercept, and x and y are coordinate points.

Quadratic functions always from a **parabola** when they are graphed, opposed to straight lines. The standard form equation of a quadratic function is denoted as $\mathbf{y = ax^2 + bx + c}$, where a, b and c are constants, and $a \neq 0$.

Remember! Standard form of a quadratic equation looks like:

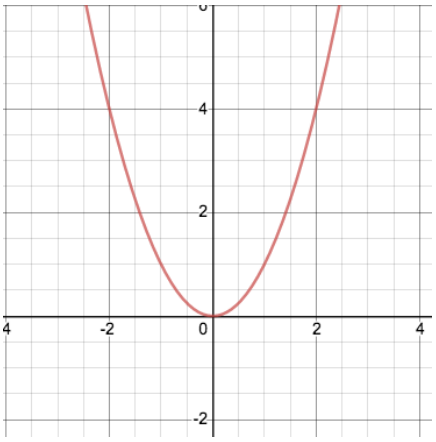
$y = ax^2 + bx + c$

Some example of quadratic functions include:

Graphs of parabolas:

1. $y = x^2$ ← Degree of 2

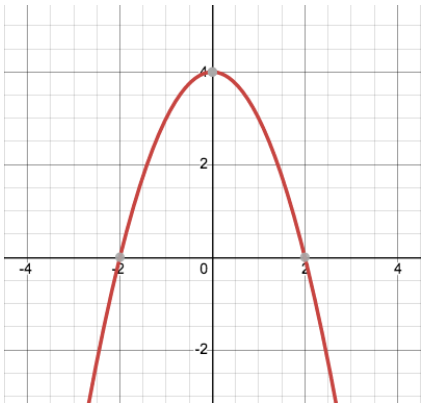
a value = 1
b value = 0 (not in equation)
c value = 0 (not in equation)



Degree of 2 →

2. $y = -x^2 + 4$

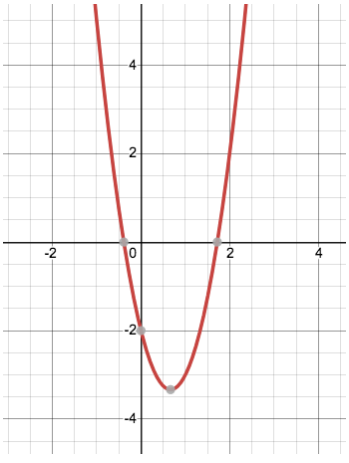
a value = -1
b value = 0 (not in equation)
c value = 4



Degree of 2 →

3. $y = 3x^2 - 4x - 2$

a value = 3
b value = - 4
c value = 2



Please watch the following video as an introduction to our next section!

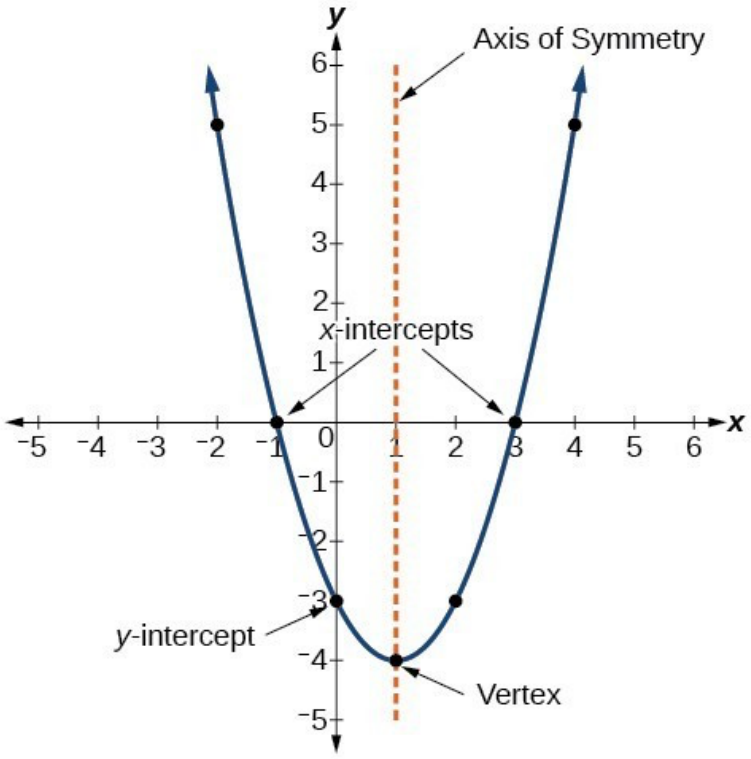
https://www.youtube.com/watch?v=BGz3pkoGPag

Section 2: Properties of parabolas

All parabolas contain the following properties that describe their "behaviour":

1. **Vertex**: given as a coordinate point, tells you the maximum or minimum point on the graph
2. **Y – intercept**: given as a coordinate point, represents where the graph crosses the y-axis (x will always be 0 at this point)
3. **X – intercept**: given as a coordinate point, represents where the graph crosses the x-axis (y will always be 0 at this point)
4. **Axis of symmetry**: the vertical line on the parabola that cuts the graph perfectly in half. This is always the same as the x coordinate of the vertex.
5. **Domain**: the set of all the valid x values for the graph
6. **Range:** the set of all the valid y values on the graph

Some of these properties are displayed in the image below:

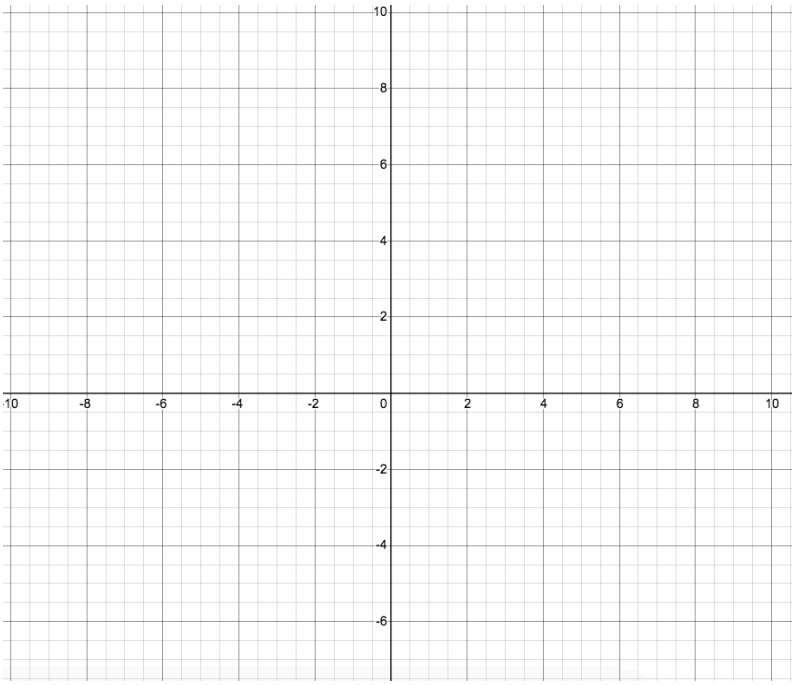


We will cover this in greater detail next week.

Section 3: Graphing parabolas

Part 1. Use the following website to graph each of the four quadratic functions below on the same graph: https://www.desmos.com/calculator

1. $y = 1x^2 + 2x - 3$
2. $y = -2x^2 + 2x - 3$
3. $y = -0.5x^2 + 2x - 3$
4. $y = 3x^2 + 2x - 3$



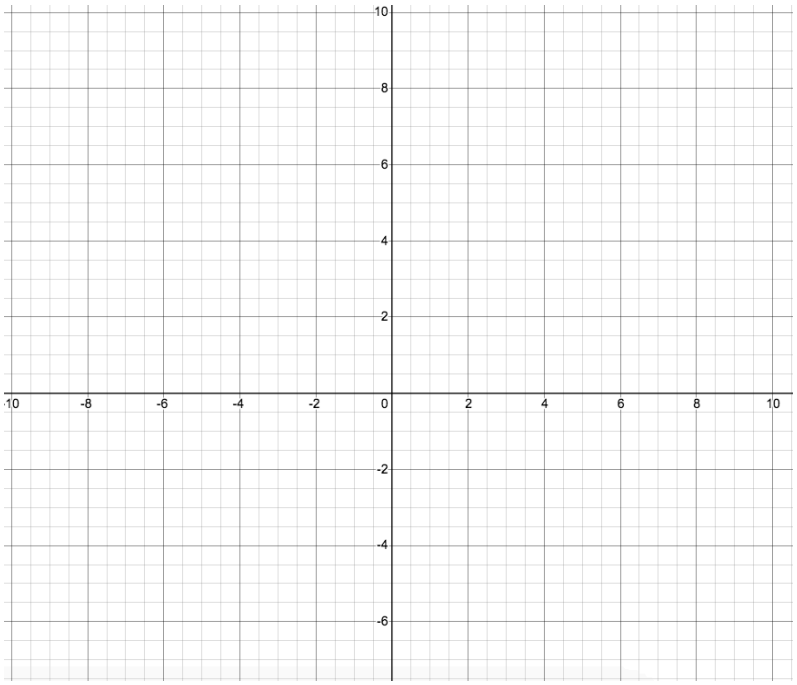
Notice that in each of the four equations above, the only constant that has been changed is the a constant. Constants b and c remain the same. Take a look at how this impacts the graphs you have created on desmos.

We can see that the value of the 'a' constant in our quadratic equation $y = ax^2 + bx + c$ controls two things:

1. The direction of opening for the parabola
    a. positive = opens up
    b. negative = opens down
2. The thickness of the parabola
    a. When a is greater than zero but less than 1, the parabola is thicker
    b. When a is greater than 1, the parabola becomes thinner

Part 2. Use the desmos website again to help you in plotting the following quadratics.

1. $y = x^2 + 3x + 1$
2. $y = x^2 + 2x + 1$
3. $y = x^2 - 5x + 1$
4. $y = x^2 - 4x + 1$



Notice that in each of the four equations above, the only constant that has been changed is the b constant. Constants a and c remain the same. Take a look at how this impacts the graphs you have created on desmos.

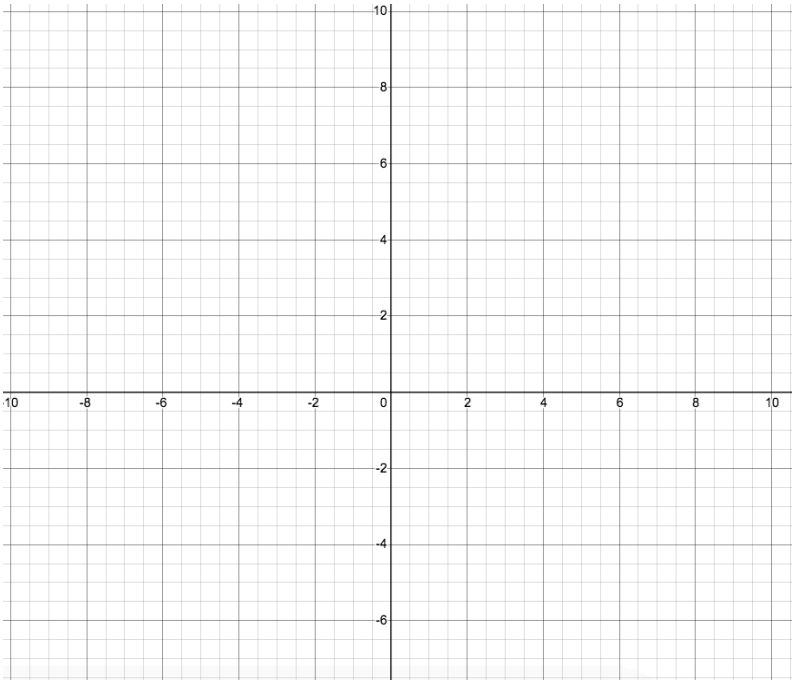
We can see that the value of the 'b' constant in our quadratic equation $y = ax^2 + bx + c$ controls two things:

1. The position of the parabola side to side
    a. When b is greater than 0 = moves left
    b. When b is less than zero = moves right
2. The vertical position of the parabola

Part 3. Use the desmos website again to help you in plotting the following quadratics.

1. $y = 2x^2 - 4x + 1$
2. $y = 2x^2 - 4x + 3$
3. $y = 2x^2 - 4x - 2$
4. $y = 2x^2 - 4x$

Hint: no c value present in equation indicates that the value is zero. It does not mean that there is no c value present at all



Notice that in each of the four equations above, the only constant that has been changed is the c constant. Constants a and b remain the same. Take a look at how this impacts the graphs you have created on desmos.

We can see that the value of the 'c' constant in our quadratic equation $y = ax^2 + bx + c$ controls one thing:

1. The position of the parabola vertically

In fact, the c value of your quadratic equation will ALWAYS correspond to the y-intercept on your parabola

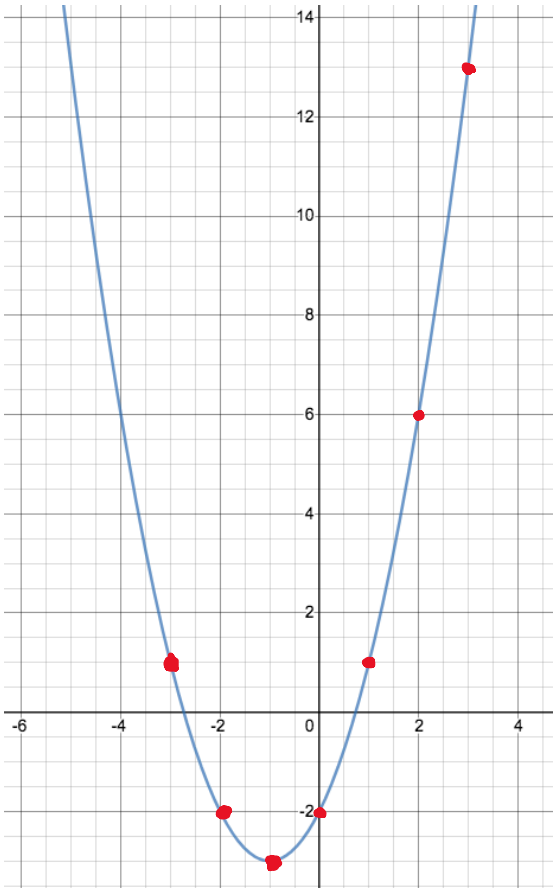
So what happens when you need to graph a quadratic equation in standard form, but you don't have a handy graphing calculator or desmos application? You set up a table of values!

Example 1. Graph the parabola of the quadratic equation $y = x^2 + 2x - 2$ using a table of values.

X point	Substitution	Y point
-3	$y = (-3)^2 + 2(-3) - 2$	1
-2	$y = (-2)^2 + 2(-2) - 2$	-2
-1	$y = (-1)^2 + 2(-1) - 2$	-3
0	$y = (0)^2 + 2(0) - 2$	-2
1	$y = (1)^2 + 2(1) - 2$	1
2	$y = (2)^2 + 2(2) - 2$	6
3	$y = (3)^2 + 2(3) - 2$	13

1. Plot your coordinate pairs (x, y):
2. Connect your points
3. Sketch you parabola

(-3, 1)
(-2, -2)
(-1, -3)
(0, -2)
(1, 1)
(2, 6)
(3, 13)



The following video summarizes this concept well:

https://www.youtube.com/watch?v=MDppAkE7UOs

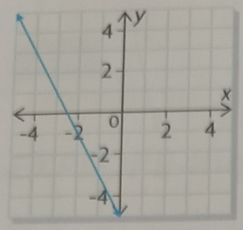
Please complete the following questions as practice problems. I will post the answers to the problems on Thursday, and take any of your questions during the zoom meeting on Wednesday.

Practice

FURTHER Your Understanding

1. Which graphs appear to represent quadratic relations? Explain.

a)



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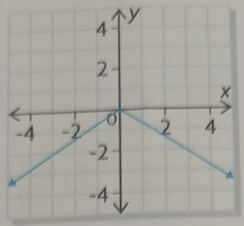

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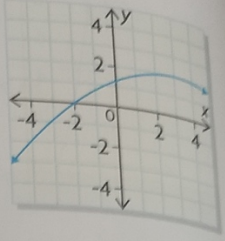
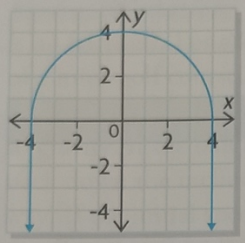
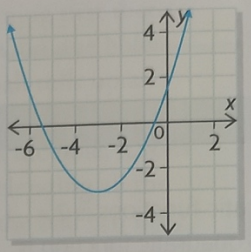


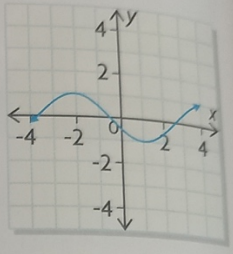
b)



d)



f)



2. Which of the following relations are quadratic? Explain.

a) $y = 2x - 7$ **d)** $y = x^2 - 5x - 6$

b) $y = 2x(x + 3)$ **e)** $y = 4x^3 + x^2 - x$

c) $y = (x + 4)^2 + 1$ **f)** $y = x(x + 1)^2 - 7$

3. State the y-intercept for each quadratic relation in question 2.

4. Explain why the condition $a \neq 0$ must be stated when defining the standard form, $y = ax^2 + bx + c$.

5. Each of the following quadratic functions can be represented by a parabola. Does the parabola open up or down? Explain how you know.

a) $y = x^2 - 4$ **c)** $y = 9 - x + 3x^2$

b) $y = -2x^2 + 6x$ **d)** $y = -\frac{2}{3}x^2 - 6x + 1$

6. Each table of values lists points in a quadratic relation. Decide, without graphing, the direction in which the parabola opens.

a)

x	-4	-3	-2	-1	0	1
y	12	5	0	-3	-4	-3

b)

x	0	1	2	3	4	5
y	-13	-3	3	5	3	-3

c)

x	-5	-4	-3	-2	-1	0
y	3.0	-0.5	-3.0	-4.5	-5.0	-4.5

d)

x	0	1	2	3	4	5
y	-4	19	40	59	76	91

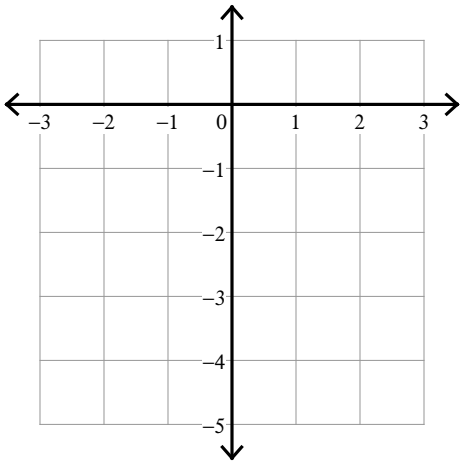
 Name_____

Graphing Quadratic Functions

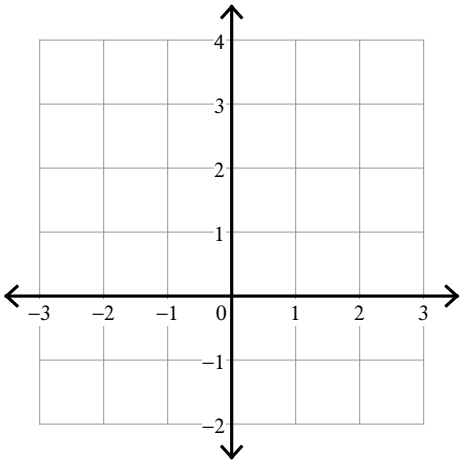
Date_____ Period____

Using a table of values, sketch a graph of each of the following equations. For each one, determine the:
a) Vertex ~~b) Domain~~ ~~c) Range~~ d) Equation of the axis of symmetry e) y-Intercept

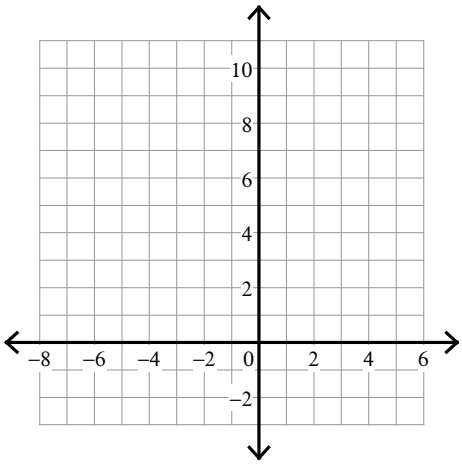
1) $y = -x^2$



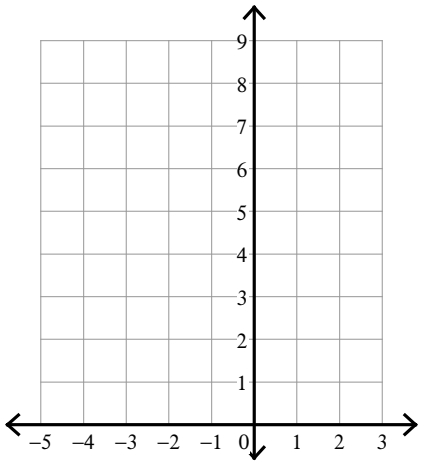
2) $y = \frac{1}{2}x^2$



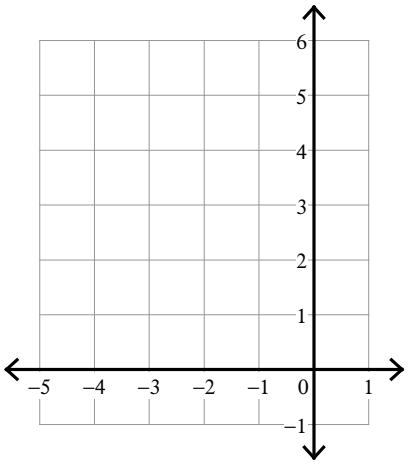
3) $y = 3x^2 + 6x + 1$



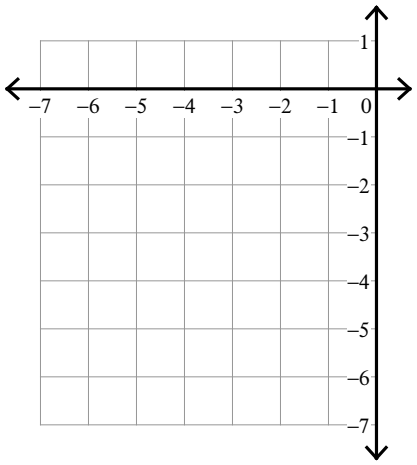
4) $y = x^2 + 2x + 5$



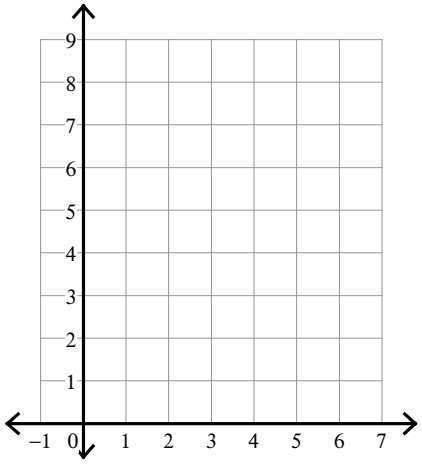
5) $y = \frac{1}{2}x^2 + 2x + 4$



6) $y = -x^2 - 8x - 17$



7) $y = (x-3)^2 + 4$



8) $y = 2(x-4)^2 + 1$