1.1 Simple Interest p.6

Name ________________

Date ________________

Goal: Solve problems that involve simple interest.

1. **term**: The contracted duration of an investment or loan.

2. **interest**: The amount of money earned on an investment or paid on a loan.

3. **fixed interest rate**: An interest rate that is guaranteed not to change during the term of an investment or loan.

4. **principal**: The original amount of money invested or loaned.

5. **simple interest**: The amount of interest earned on an investment or paid ona loan based on the original amount (the principal) and the simple interest rate.

6. **maturity:** The contracted end date of an investment or loan, at the end of the term.

7. **rate of return:** The ratio of money earned (or lost) on an investment relative to the amount of money invested, usually expressed as a decimal or a percent.

INVESTIGATE the math

Sera is 20 years old and needs money to pay for college. When she was born, her grandparents bought her a $500 Canada Savings Bond (CSB) with a **term** of 10 years. They chose a CSB as an investment because they liked the security of loaning money to the government. The **interest** earned was determined using a **fixed interest rate** of 6% per year on the original investment and was paid at the end of each year until Sera's 10th birthday.

How can you determine the current value of Sera's CSB?

A. How much interest was earned on the **principal** by the end of the first year?

$i = Prt$
$= 500(0.06)(1)$
$= \$30$

B. Determine the **simple interest** earned each year, the accumulated interest, and the value of the investment for the first 4 years. Organize your calculations in a table.

Year	Value of Investment @ Start of Year ($)	Simple Interest Earned Each Year ($)	Accum. Interest ($)	Value of Investment @ End of Year ($)
0				500
1	500	$30	$30	$ 530
2	$530	$30	$60	$ 560
3	$560	$30	$90	$ 590
4	$590	$ 30	$ 120	$ 620

C. Is the simple interest earned each year constant or variable? Explain.

doesn't change, ∴ constant

D. Describe the relationship between the number of years, the interest earned each year, and the accumulated interest.

Interest earned each year multiplied by the number of years is the accumulated intere

Example 1: Solving a simple interest problem (p.8)

Marty invested in a $2500 guaranteed investment certificate (GIC) at 2.5% simple interest, paid annually, with a term of 10 years.

a) How much interest will accumulate over the term of Marty's investment? Use the formula $i = Prt$, where i = interest, P = the principal, r = rate as a decimal and t = time

$i = (\$2500)(0.025)(10)$

$= \$625$

b) What is the **future value** of his investment at maturity?

future value = A

$A = P + i$

$A = P + Prt$

$\$2500 + \$625$

$\$3125$

c) Use Marty's investment to write an algebraic expression that could be used to determine the future value of any investment earning simple interest.

$A = P + Prt$

$A = P(1 + rt)$

Example 3: Determining the duration of a simple interest investment (p. 10)

Ingrid invested her summer earnings of $5000 at 8% simple interest, paid annually. She intends to use the money in a few years to take a holiday with a girlfriend.

a) How long will it take for the future value of the investment to grow to $8000?

$A = P + Prt$

$P = \$5000$

$r = 0.08$

$A = \$8000$

$t = ?$

$8000 = 5000 + 5000(0.08)t$

$8000 = 5000 + 400t$

$\frac{3000}{400} = \frac{400t}{400}$

$7.5 = t$

It will take 7.5 years* for the future value to equal $8000

* since interest is paid annually, we should round our answer to 8 years

b) What is Ingrid's **rate of return** ?

$A = P(1 + rt)$

$A = 5000(1 + 0.08(8))$

$= 5000(1.64)$

$= \$8200$

$\text{Rate of Return} = \frac{i}{P}$; i = interest earned, P = principal

$\frac{3200}{5000} = 0.64 \rightarrow 64\%$

The rate of return over 8 years is 64%

Example #4: Determining the rate of interest on a simple interest investment (p.12)

Grant invested $25 000 in a simple interest Canada Savings Bond (CSB) that paid interest annually.

a) IF the future value of the CSB is $29 375 at the end of 5 years, what interest rate does the CSB earn?

$A = P + Prt$

$P = 25\,000$
$A = 29\,375$
$t = 5$
$r = ?$

$29\,375 = 25\,000 + 25\,000r(5)$

$\frac{4\,375}{125\,000} = \frac{125\,000r}{125\,000}$

$0.035 = r$

The interest rate is 3.5% per annum

b) Grant cashed in the bond after 4.5 years because a house he had been admiring came up for sale and he needed a down payment. How much money did he have for the down payment?

$A = ?$
$P = 25\,000$
$r = 0.035$
$t = 4$

$A = P(1 + rt)$
$A = 25\,000(1 + 0.035(4))$
$A = 25\,000(1.14)$
$= 28\,500$

Grant had $28 500 for the down payment

F Math 12

1.2 Exploring Compound Interest p.8

Name ______________

Date ______________

Goal: Compare simple interest with compound interest.

1. **compound interest**: The interest that is earned or paid on both the principal and the accumulated interest.

Explore the math

Guaranteed investment certificates (GICs) can earn either simple or compound interest. If a GIC earns simple interest annually, the same amount of interest is earned every year. If a GIC earns **compound interest** annually, the interest at the end of the first year is earned on the principal, but the interest at the end of the second year is earned on the principal plus the interest from the first year. Each year after that, the interest is earned on the principal plus all the accumulated interest from the previous years.

Both Ewan and Rena received a $1000 prize in a story-writing contest.

- Ewan bought a $1000 simple interest GIC with his prize money. It has a 5-year term and earns 3.6% paid annually.
- Rena bought a $1000 compound interest GIC with her prize money. It also has a 5-year term and earns 3.6% paid annually.

How do the future values of Ewan's and Rena's investments compare at maturity?

A. With a partner, compare your answers and the strategies you used to determine the difference between the two investments at maturity.

- Use $A = P(1 + rt)$ to determine the future value of Ewan's investment

$$A = 1000(1 + (0.036)(5))$$
$$= \$1180$$

P

C. How much would Ewan need to invest at 3.6% simple interest to earn the same as Rena in 5 years?

$$A = P + Prt \rightarrow A = P(1 + rt)$$

$$1193.44 = P(1 + (0.036)(5))$$

$$\frac{1193.44}{1.18} = \frac{P(1.18)}{1.18}$$

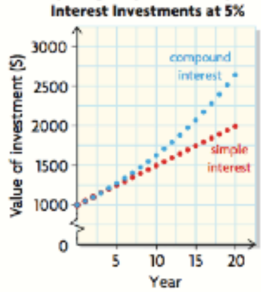
$P = \$1011.39$

In Summary

Key Ideas

- Compound interest is determined by applying the interest rate to the sum of the principal and any accumulated interest. Previously earned interest is reinvested over the course of the investment.
- If the same principal is invested in a compound interest account and a simple interest account, with the same interest rate for the same term, the compound interest investment will grow faster (non-linear) than the simple interest investment (linear). For example, the graphs show principal of $1000 invested over 20 years at 5% simple interest (red graph) and 5% compound interest (blue graph), both paid annually.

Comparing Simple and Compound Interest Investments at 5%



Need to Know

- Financial institutions pay compound interest on investments at regular equal intervals. If interest is paid annually, it is calculated at the end of the first year on the principal and then added to the principal. At the end of the second year, the interest is calculated on the balance at the end of the first year (principal plus interest earned from the previous year). This pattern continues every year until the end of the investment term.

HW: 1.2 p. 19 #1-3

September 14, 2012

F Math 12

1.3 Compound Interest: Future Value p. 20

Name ______________

Date ______________

Goal: Determine the future value of an investment that earns compound interest.

1. **compounded annually**: When compound interest is determined or paid yearly.

   ↘ interest is added to the principal for the future interest calcs.

2. **compounding period:** The time over which interest is determined; interest can be compounded annually, semi-annually (every 6 months), quarterly (every 3 months), monthly, weekly, or daily.

3. **Rule of 72**: A simple formula for estimating the doubling time of an investment; 72 is divided by the annual interest rate as a percent to estimate the doubling time of an investment in years. The Rule of 72 is most accurate when the interest is compounded annually.

LEARN ABOUT the math

Yvonne earned $4300 in overtime on a carpentry job. She invested the money in a 10-year Canada Savings Bond that will earn 3.8% **compounded annually** . She decided to invest in a CSB, instead of keeping the moneyin a savings account, because the CSB will earn more interest.

What is the future value of Yvonne's investment after 10 years?

Example 1: Using reasoning to develop the compound interest formula (p.20)

$$A = P + Prt$$
$$= P(1+rt)$$

$$A_1 = 4300(1 + (0.038)(1))$$
$$= 4300(1.038)$$
$$= \$4\,463.40$$

$$A_2 = 4\,463.40(1 + (0.038)(1))$$
$$= 4\,463.40(1.038)$$
$$= \$4\,633.01$$

Example 2: Determining the future value of an investment with semi-annual compounding (p. 22)

Matt has invested a $23 000 inheritance in an account that earns 13.6%, compounded semi-annually. The interest rate is fixed for 10 years. Matt plans to use the money for a down payment on a house in 5 to 10 years. (2 times a year)

a) What is the future value of the investment after 5 years? What is the future value after 10 years?

$A = P(1+i)^n$

$A_5 = 23000\left(1 + \frac{0.136}{2}\right)^{10}$

$= 23000(1 + 0.068)^{10}$

$= 23000(1.068)^{10}$

$= \$44\,405.87$

$A_{10} = 23000\left(1 + \frac{0.136}{2}\right)^{20}$

$= 23000(1.068)^{20}$

$= \$85\,733.96$

b) Compare the principal and the future values at 5 years and 10 years. What do you notice?

@ 5 years, principal is almost doubled

@ 10 years, principal has more than tripled

c) If the investment had earned simple interest, would the relationship between the principal and the future values have been the same? Explain.

$I_5 = Prt$

$= (23000)(0.136)(5)$

$= \$15\,640$

$I_{10} = \$31\,280$

$(I_5 \times 2)$

simple interest only pays interest on the principal, so the future value is less

F Math 12 **_1.5 Investments Involving Regular Payments p. 46_**

Name ________________

Date ________________

Goal: Determine the future value of an investment that earns compound interest involving regular payments.

THESE TYPES OF PROBLEMS CAN ONLY BE SOLVED USING THE TVM SOLVER OR A SPREADSHEET OR BY DOING A LOT OF REPETATIVE CALCULATIONS BY HAND!

INVESTIGATE the math

Pokiak is now 18 years old, and he needs money for his post-secondary education. On his 14th birthday, his family deposited $1000 into a Registered Education Savings Plan (RESP) at 3% interest, compounded annually. Since then, Pokiak has deposited $1000 of his own money, earned by working part-time, into the account each year.

How much money is in Pokiak's RESP account, and how much interest has it earned altogether?

N = 4

I% = 3%

PV = -1000

PMT = -1000

FV = ?

P/Y = 1

C/Y = 1

A = \$5309.14

I = A − all deposited monies

= 5309.14 − 5000

= \$309.14

Age → 14 $\underline{\ \$1k\ }$ 15 $\underline{\ \$1k\ }$ 16 $\underline{\ \$1K\ }$ 17 $\underline{\ \$1K\ }$ 18

\$1k

Example 1: Determining the future value of an investment involving regular deposits (p.47)

Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits $500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8%, compounded semi-annually. How much money will be in the account at the end of 5 years? How much of this money will be earned interest?

N = 5 × 2

I% = 3.8

PV = 0

PMT = −500

FV = ?

P/Y = 2

C/Y = 2

A = $5449.90

How much did she deposit?

$500 × 10 = $5000

I = A − deposits

= $5449.90 − $5000.00

= $449.90

F Math 12

2.4 Buy, Rent or Lease p. 120

Name

Date

Goal: Solve problems by analyzing renting, leasing, and buying options.

1. **lease**: A contract for purchasing the use of property, such as a building or vehicle, from another, the lessor, for a specified period.
2. **equity**: The difference between the value of an item and the amount still owing on it; can be thought of as the portion owned. For example, if a $25 000 down payment is made on a $230 000 home, $205 000 is still owing and $25 000 is the equity or portion owned.
3. **asset**: An item or a portion of an item owned; also known as property. Assets include such items as real estate, investment portfolios, vehicles, art, and gems.
4. **appreciation:** increase in the value of an asset over time.
5. **depreciation:** Decrease in the value of an asset over time.
6. **disposable income:** The amount of income that someone has available to spend after all regular expenses and taxes have been deducted.

$$A = P(1-R)^n$$

Depreciation Formula

where A = future value

P = present value (principal)

R = depreciation rate

n = number of depreciating periods

LEARN ABOUT the Math

Amanda is a civil engineer. She needs a vehicle for work, on average, 12 days each month. She has been renting a vehicle when she needs it.The advantage to renting is that she simply fills the gas tank and drops off the vehicle when she is done with it. The disadvantage is that she has to spend time arranging for the rental, picking up the vehicle, and getting home after dropping it off. She is wondering if renting is the most economical choice and is considering her options:

- She could **lease** a vehicle, which requires a down payment of $4000 and lease payments of $380 per month plus tax. She would need insurance at $1220 each year (which could be paid monthly) and would have to pay for repairs and some maintenance, which would average $50 each month. For the 4-year lease she is looking at, she would have no **equity** in the vehicle at the end of the term, since the car would belong to the leasing company.
- She could buy a vehicle for $32 800 and finance it for a 4-year term at 4.5% interest, compounded monthly. She would have the same insurance, repair, and maintenance costs that she would have with leasing. However, the equity of the vehicle would be considered an **asset.**
- She could continue to rent at $49.99 per day, plus tax, with unlimited kilometres.

Which option would you recommend for Amanda, and why?

Example 1: Solving a problem that involves leasing, buying, or renting a vehicle (p.121)

Figure out the monthly cost for the three options listed above.

Lease

$$\underbrace{\$4000}_{\text{down payment}} + \underbrace{\$380 \times 12 \times 4}_{\text{monthly payment}} + \underbrace{\$1220 \times 4}_{\text{insurance}} + \underbrace{\$50 \times 12 \times 4}_{\text{maintenance}}$$

$29 520 → total cost of leasing for 4 years

$$\frac{\$29\,520}{12 \cdot 4} = \$615 \text{ per month}$$

Buying

N = 4 x 12

I% = 4.5

PV = 32 800

* PMT = -747.95 → monthly payment

FV = 0

P/Y = 12

C/Y = 12

$$\underbrace{\$747.95}_{\text{payment}} + \underbrace{\frac{\$1220}{12}}_{\text{insurance}} + \underbrace{\$50}_{\text{repairs}}$$

$899.62

Renting

$49.99 x 12 = $599.88

Leasing: $615 Buying: $899.62 Renting: $599.88

Recommendation would vary based on need

Example 2: Solving a problem that involves vehicle depreciation (p.122)

A luxury vehicle rental company depreciates the value of its vehicles each year over 5 years. At the end of the fifth year, the company writes off a vehicle for its scrap value. The company uses a depreciation rate of 40% a year.

a) What is the scrap value of each car below?
   i) Car A, which is currently 2 years old and has a value of $43 200
   ii) Car B, which is currently 1 year old and has a value of $75 600

b) What was the original purchase price of each car?

$A = P(1-R)^n$

a) i) $A = 43200(1-0.40)^3$ → $5-2$

scrap age ↑ ↑ current age

$= \$9\,331.20$

ii) $A = 75\,600(1-0.40)^4$

$= \$9\,797.76$

Car A

b) $\dfrac{43200}{(1-0.40)^2} = \dfrac{P\cancel{(1-0.40)^2}}{\cancel{(1-0.40)^2}}$

$\dfrac{43200}{0.60^2} = P$

$\$120\,000 = P$

Car B

$\dfrac{75\,600}{1-0.40} = \dfrac{P(1-0.40)^1}{1-0.40}$

$\dfrac{75\,600}{0.60} = P$

$\$126\,000 = P$

Example 3: Solving a problem that involves leasing or buying a water heater (p. 124)

The 10-year-old hot water heater in Tom's home stopped working, so he needs a new one. Tom works for minimum wage. After paying his monthly expenses, he has $35 **disposable income** left. He has an unused credit card that charges 18.7%, compounded daily. He has two options:

- Tom could lease from his utility company for $17.25 per month. This would include parts and service.
- He could buy a water heater for $712.99, plus an installation fee of $250, using his credit card. He could afford to pay no more than $35 each month.

a) What costs are associated with buying and leasing?

b) What do you recommend for Tom? Justify your recommendation.

c) Suppose that the life expectancy of a water heater is 8 years. Would this change your recommendation?

Buying on CC

* N: 36.307... → 37
I%: 18.7
PV: 712.99 + 250
PMT: −35
FV: 0
P/Y: 12
C/Y 365

total cost: N · PMT

$1270.76

Lease

$17.25 x 37

$638.25

the lease company would cover repairs

b) It makes more sense to lease – lower cost, no repair cost

c) $ 17.25 x 12 x 8

$1656

total lease cost exceeds purchase price, ∴ he should buy

Example 5: Solving a problem that involves renting or buying a house (p. 127)

Two couples made different decisions about whether to rent or buy:

a) Helen and Tim bought a house for $249 900. They have negotiated a mortgage of 95% of the purchase price, so they will need a 5% down payment. The mortgage is compounded semi-annually at 5.5%, has a 20-year term, and requires monthly payments.

b) Don and Pat are renting a house for $1600 per month. They plan to renew the lease yearly. After 3 years, both couples decide to move. Helen and Tim discover that the value of their house has depreciated by 10% over the 3 years.

Compare each couple's situation after 3 years.

a) $\$249\,900 - \underbrace{\$12\,495}_{\text{down payment}}$

$237 405

N: 20×12
I%: 5.5
PV: 237 405
* PMT: −1624.78
FV: 0
P/Y: 12
C/Y: 2

monthly mortgage payment is $1624.78

Renting

b) $1600 × 12 × 3

$57 600

Cost of renting

N: 3×12
I%: 5.5
PV: 237 405
PMT: −1624.78
* FV: −215 992.56
P/Y: 12
C/Y: 2

to find out how much is left on the mortgage

depreciation

1 − R

new value of the house: $249 900(0.90)

∴ $224 910

paying back the bank: 224 910 − 215 992.56

$ 8 917.44

profit from selling the house

Cost of owning: $\$12\,495 + \underbrace{3 \times 12 \times 1624.78}_{\text{mortgage payments}} - \underbrace{8\,917.44}_{\text{profit from selling}}$

down payment

$ 62 069.64

The renters spent $57 600, the owners spent $62 069.64

1.1 Making Conjectures: Inductive Reasoning

If the same result occurs over and over again, we may conclude that it will always occur. This kind of reasoning is called **inductive reasoning.**

Inductive reasoning can lead to a **conjecture**, which is a testable expression that is based on available evidence but is not yet proved.
(doesn't have to be true, just that it can be tested)

Example 1: Use inductive reasoning to make a conjecture about the product of an odd integer and an even integer.

Step 1: Come up with some available evidence:

$7 \times 6 = 42$
$3 \times 10 = 30$
$(9)(12) = 108$

some possible conjectures

Conjecture(s):

The product of an odd integer & an even integer:

- Is Even

OR - Is Positive

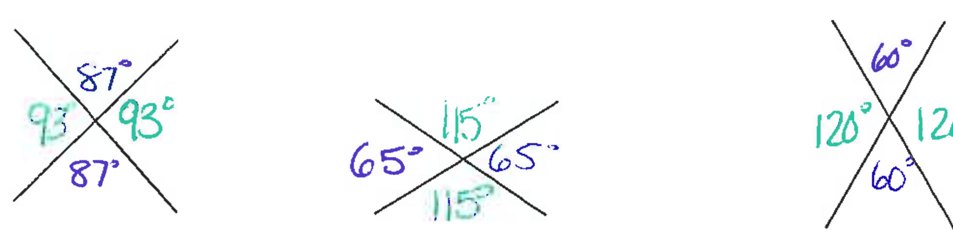
OR - Is more than one digit ✗

OR - Is divisible by 3.

but if $(3)(2) = 6$ is our evidence we can't have this one

Example 2: Make a conjecture about intersecting lines and the angles formed.

Evidence:



Conjecture(s):

- Opposite angles of intersecting lines are equal

OR - Two adjacent angles of intersecting lines add to 180°

OR - All angles of intersecting lines add to 360°

Example 3: Make a conjecture about the sum of two odd numbers.

Evidence:
(some examples)

$7+3=10$
$7+13=20$
$5+3=8$
$-3+7=4$
$-21+-3=-24$

Based on this evidence.

Conjecture:
The sum of two odd numbers is even.

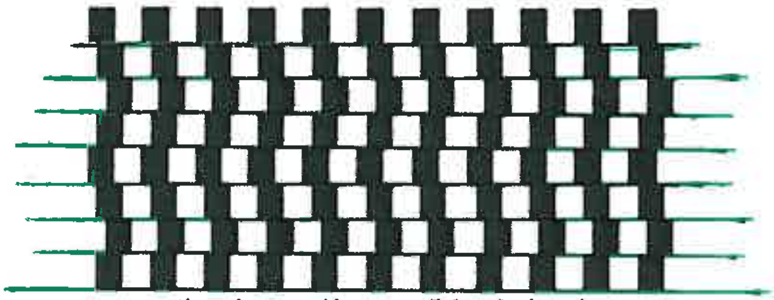
Assignment: pg. 12 #3, 5, 6, 9, 10-12, 14, 16, 20

FOM 11 **1.2 Exploring the Validity Of Conjectures**

Some conjectures initially seem to be valid, but are shown not to be valid after more evidence is gathered.

Example 1: Make a conjecture about the lines below:

Conjecture: The horizontal lines are not parallel.



Are these horizintal lines parallel or do they slope?

When checked, this conjecture is invalid (not true). The lines are parallel.

Example 2: Make a conjecture about the grey rectangles:

Conjecture: The rectangles on top are darker than the rectangles on the bottom.

The rectangles are in fact the same colour.

The best we can say about a conjecture reached through inductive reasoning is that there is **evidence either to support or deny it**.

Assignment: pg. 17 #1-3

FOM 11 **1.3 Using Reasoning to Find A Counterexample to a Conjecture**

We know that inductive reasoning can lead to a conjecture, which may or may not be true. One way a conjecture may be proven false is by a **counterexample**.

Example 1: If possible, find a counterexample for each conjecture. If not, write "true".

a. Conjecture: Every mammal has fur.

Counterexample: Whales

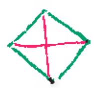
b. Conjecture: The acute angles in a right triangle are equal.

Counterexamble 

c. Conjecture: A polygon has more sides than diagonals.

Try Some:

3sides 0diagonals 

 4sides 2 diagonals

 5 sides 5diagonals

 6sides 9diagonals counterexamples

d. Conjecture: The square of every even number is even.

$0^2=0$ $2^2=4$ $(-4)^2=16$ TRUE

e. Conjecture: An even number is any number which is not odd.

Counterexample. $\frac{1}{2}$ (OR 0.5) not odd, but not even either.

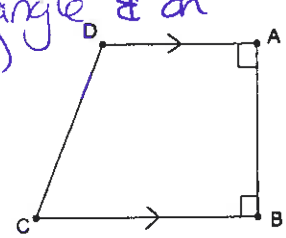
Example 2: Three conjectures are given.
For which conjectures is this diagram a counterexample?

A. The opposite sides of a parallelogram are equal.

Not a counter example (the diagram is not a parllelogram)

B. A quadrilateral cannot have both a 90° angle and an obtuse angle.

✓ Counterexample. (the diagram is a quadrilateral with a 90° angle & an obtuse angle.)



C. Every trapezoid has 2 pairs of equal angles.

✓ Counterexample (the diagram is a trapezoid without 2 pairs of equal angles)

Assignment: pg. 22 #1, 3-6, 10, 12, 14, 17

When we make a conclusion based on statements that we accept as true, we are using **deductive reasoning**.

↳ usually involves algebra.

Example 1: Use deductive reasoning to prove that the product of an odd integer and an even integer is even.

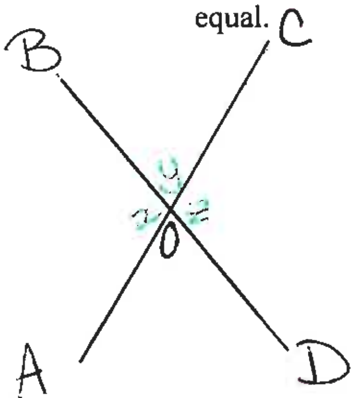
Let the even integer be $\boxed{2x}$ (x is an integer)

& the odd integer be $\boxed{2y+1}$ (y is an integer)

$$\Rightarrow \text{odd} \times \text{even} = (2y+1)(2x)$$
$$= 4xy + 2x$$
$$= 2(2xy + x)$$
$$= 2(\text{some } \#)$$

2 times any number will always be even.

Example 2: Use deductive reasoning to prove that opposite angles of intersecting lines are equal.



$\angle AOC = 180°$	Straight Line
$x + y = 180°$	∠'s on straight line add to 180°
$\angle BOD = 180°$	Straight Line
$z + y = 180°$	∠'s on straight line add to 180°
$x + y = z + y$	both = 180°
$x = z$	Subtract 'y' from both sides

↳ x & z are opposite angles and they will always be equal

subtract

one number & the one right after it

$x, x+1, x+2,$ etc.

Example 3: Use deductive reasoning to prove that the difference between consecutive perfect squares is always an odd number.

$x \cdot x = x^2$

Let x & $x+1$ be our consecutive numbers

Prove: $(x+1)^2 - x^2$ will always be odd.

	x	$+1$
x	x^2	$+x$
$+1$	$+x$	$+1$

$$(x+1)^2 - x^2$$
$$= x^2 + x + x + 1 - x^2$$
$$= x^2 - x^2 + 2x + 1$$
$$= 2x + 1$$

this will always be odd because $2x$ is always even & adding 1 makes it odd.

Example 4: Weight-lifting builds muscle. Muscle makes you strong. Strength improves balance. Inez lifts weights. What can be deduced about Inez?

Inez → Weight Lifting → Builds Muscle → Strength → Improved Balance

$\boxed{\therefore \text{ Inez builds muscle, improving his strength \& balance}}$

Assignment: pg. 31 #1, 2, 4-7, 10, 11, 15, 19

FOM 11 1.4.2 Deductive Reasoning Part II

When we make a conclusion based on statements that we accept as true, we are using **deductive reasoning**. The rules we follow when performing algebraic manipulations are things that we accept (and know) as true. So we are using deductive reasoning to prove a statement is always true.

Statements that we know are true:

Any integer multiplied by 2 is an even number.

- This means that $2x$ or 2(any combination of variables and coefficients) will **always** be even.

If you add 1 to any even integer you will get an odd number.

- This means that $2x + 1$ or 2(any combination of variables and coefficients) + 1 will **always** be odd.

Consecutive Numbers follow each other in numerical order

- This means that x, $x + 1$, $x + 2$, $x + 3$ are 4 numbers that come one after the other numerically.
- $2x$, $2x + 2$, $2x + 4$, $2x + 6$ are 4 consecutive even numbers
- $2x + 1$, $2x + 3$, $2x + 5$, $2x + 7$ are 4 consecutive odd numbers

Example 1: Use deductive reasoning to prove that the sum of an odd number and an even number is always odd.

$2x+1$

Add

$2y$

$$(2x+1) + (2y)$$
$$= 2x+1+2y$$
$$= 2x+2y+1$$
$$= \underbrace{2(x+y)}_{\text{this is even}}+1 \rightarrow$$

An even number +1 will always be odd!

so $2(x+y)+1$ is odd!

so An odd # + even # will always be odd.

Finishing a Proof:

- If proving an answer is even it should look like this → 2(any combination of variable terms)
- If proving an answer is odd it should look like this → 2(any combination of variable terms) + 1
- If proving an answer is divisible by 3 it should look like this → 3(any combination of variable terms)
- If proving an answer is divisible by 4 it should look like this → 4(any combination of variable terms)
- If proving an answer is divisible by 5 it should look like this → 5(any combination of variable terms)
- etc……

$(2x)^2$ $\quad$ $2x$

Example 2: Prove that the square of an even integer is always even

$$(2x)^2$$
$$= 2^2x^2$$
$$= 4x^2$$
$$= 2(2x^2)$$

multiplied by 2 $\Rightarrow$ so it will always be even!

Example 3: Prove that the result of the number trick below is always the number you start with.

- Choose a number $\quad x$
- Add 2 $\quad x+2 = x+2$
- Multiply by 3 $\quad 3(x+2) = 3x+6$
- Subtract 6 $\quad 3x+6-6 = 3x$
- Subtract your original number $\quad 3x - x = 2x$
- Divide by 2 $\quad 2x \div 2 = x$

Starting Number!

Example 4: The sum of a two digit number and its reversal is a multiple of 11.

Reversal of digits: xy $\quad$ & $\quad yx$

$= 10x + y \qquad = 10y + x$

Sum of reversed $\Rightarrow$ $10x + y + 10y + x$

$$= 10x + x + y + 10y$$
$$= 11x + 11y$$
$$= 11(x+y)$$

being multiplied by 11 so $11(x+y)$ is a multiple of 11!

Assignment: Deductive Reasoning Worksheet

A single error in a deductive proof will make it invalid. Some common errors are:

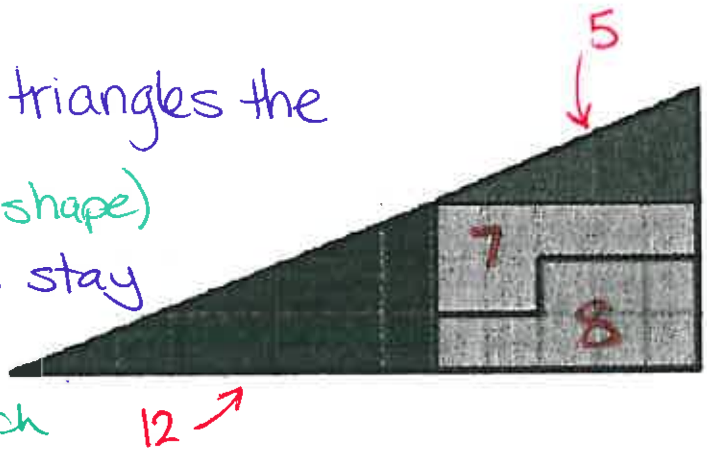
- Dividing by zero. (error in calculation)
- Circular reasoning. (starting with a false assumption)
- Confusing reasoning. (error in reasoning)

Example 1:

1. Are both large triangles the same area?
(find area of each shape)

- All small shapes stay the same.

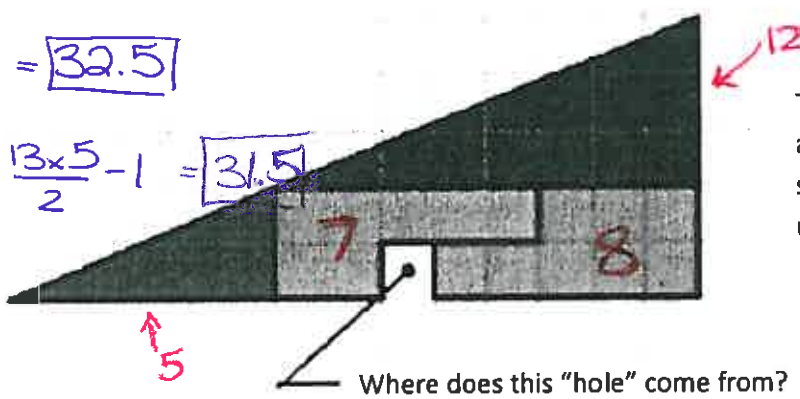
(find area of each large triangle)



Below the four parts are moved around

Top: $\frac{b \times h}{2} = \frac{13 \times 5}{2} = \boxed{32.5}$

Bottom: $\frac{b \times h}{2} - 1 = \frac{13 \times 5}{2} - 1 = \boxed{31.5}$

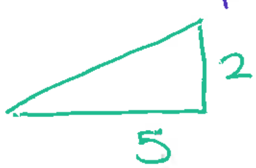


The partitions are exactly the same as those used above

How is this possible??
(made of same shapes, but different areas).

2. Find the slopes of the small triangles.

Slope $= \frac{2}{5}$



slope $= 3/8$

$\Rightarrow$ The slopes are not the same!

We made a false assumption that the large shapes were triangles (they aren't)

Example 2:

Why is this proof invalid?

Given: $a = b$

$a^2 = ab$ } $a^2 = a \cdot a \nearrow$ but $a=b$ $\searrow a^2 = ab$ ✓

$a^2 - b^2 = ab - b^2$ subtract b^2 from both sides ✓

$\boxed{(a+b)(a-b) = b(a-b)}$ factor ✓

$(a+b) = b$ divide by $(a-b)$ but since $a=b$ X $(a-b)=0$ and you cannot divide by zero!

$a + a = a$

$2a = a$

$2 = 1$!!!

$\therefore$ Error in calculation.

Example 3: Isaac claims that -3 = 3.

Proof: Assume -3 = 3. ← False Assumption (we know $-3 \neq 3$)

$$(-3)^2 = 3^2$$

$$9 = 9$$

Therefore: -3 = 3.

Where did Isaac go wrong?

Also circular Reasoning (you cannot assume the thing you are trying to prove)

Assignment: pg. 42 #1, 3, 5, 6, 7, 10

FOM 11 **1.6 Reasoning to Solve Problems**

Reminder: A **conjecture** is a conclusion based on examples.

We know that **inductive reasoning** can lead to a conjecture that may be proven by **deductive reasoning**. However, conjectures may be false, and can be disproven by a **counterexample**.

Example 1: Decide whether the process used is inductive or deductive reasoning:

a. Show the sum of two even numbers is even by using several examples. (examples usually is inductive)

$2+4=6$
$8+14=22$

INDUCTIVE

b. No mathematician is boring. Ann is a mathematician. Therefore, Ann is not boring.

(no examples → statements = deductive) DEDUCTIVE

c. One counterexample proves that a conjecture is false.

DEDUCTIVE

d. You show why your statement makes sense.

DEDUCTIVE

e. You give evidence that your statement is true.

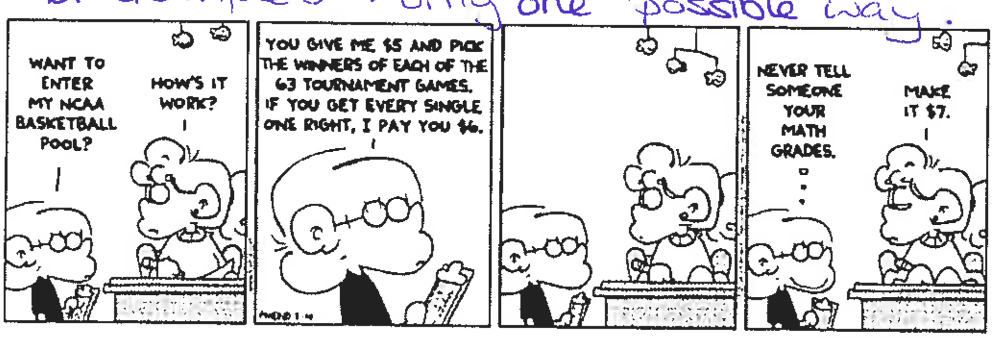
INDUCTIVE

f. Six other examples to show that your conjecture is true.

INDUCTIVE

g. What three coins have a value of $0.60?

didn't ask for examples → only one possible way. DEDUCTIVE



Draw Table to Help.

Example 2: Al, Bob, Cal, and Dave are on four sports teams.

- Each play on just one team.
- They play football, basketball, baseball, and hockey.
- Bob is a goalie. (must be hockey)
- The tallest player plays basketball, and the shortest baseball.
- Cal is taller than Dave, but shorter than Al and Bob.

What sports does each play?

	Foot	Bask.	Base	Hockey
Al	X	✓	X	X
Bob	X	X	X	✓
Cal	✓	X	X	X
Dave	X	X	✓	X

Height: DAVE; CAL; (AL → Basketball, ~~BOB~~ → Bob plays Hockey)

Short → Tall

Baseball

∴ Al plays Basketball
Bob plays Hockey
Cal plays Football
Dave plays Baseball

Example 3: Art, Bill, Cecil, and Don live in the same apartment. They are a manager, teacher, artist and musician. Art and Cecil watch TV with the teacher. Bill and Don go to the hockey game with the manager. Cecil jogs with the manager and teacher. Who is the manager?

so they can't be the teacher.

So Bill & Don not manager.

cecil not manager or teacher.

	Manager	Teacher	Artist	Musician
Art	O	X		
Bill	X			
Cecil	X	X		
Don	X			

Assignment: pg. 48 #1, 3, 5, 6, 8, 9, 10, 13, 16

∴ Art is the manager

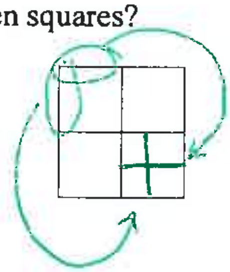
FOM 11 **1.7 Analyzing Puzzles And Games**

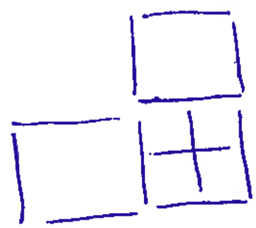
Both inductive and deductive reasoning are useful for determining a strategy to solve a puzzle or win a game.

Example 1: Use four 9's in a math equation that equals 100.

$$\boxed{99+\frac{9}{9}} = 99+(9\div 9)$$
$$= 99+1$$
$$= 100$$

Example 2: The following figure is made up of 12 sticks. Can you move just two sticks and create seven squares?





Example 3: Put the numbers 1 to 8 in each square so that each side adds to the middle term.

1	5	6
8	12	4
3	7	2

8	3	2
4	13	6
1	7	5

4	3	7
2	14	6
8	5	1

8	4	3
1	15	5
6	2	7

FOM 11 **Geometric Properties Review**

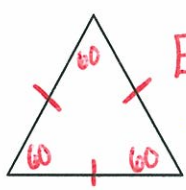
Angle Properties

θ "Theta" is symbol most commonly used for angles.

Acute $0° < \theta < 90°$	Right $\theta = 90°$	Complimentary 2 Angles add to 90° $a+b=90°$
Obtuse $90° < \theta < 180°$	Straight $\theta = 180°$	Supplementary 2 Angles add to 180° $a+b=180$
Angles on a line More than 2 Angles on line add to 180°	Reflex $180° < \theta < 360°$	Angles at a point Angles all meeting at a point add to 360° $a+b+c+d=360°$
$a+b+c+d=180°$	Vertically opposite angles opposite from each other at intersection. Angles are Equal	

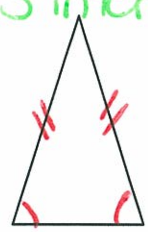
Triangle Properties

For all Triangles the 3 interior angles add to 180°.



Equilateral Triangle

- 3 equal sides
- 3 equal angles (all 60°)

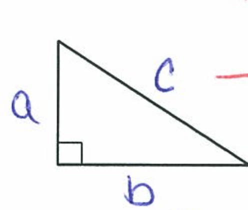


Isosceles $\triangle$

- 2 equal sides
- 2 equal angles (opposite from the equal sides)

Scalene $\triangle$

- No Equal sides
- No Equal Angles



Right $\triangle$

One Right Angle (90°)

- Can use pythagoras $a^2+b^2=c^2$

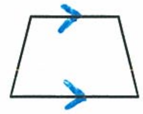
Trigonometry (Sin, Cos, Tan)

→ 4 sides (all 4 interior angles add to 360°)

Quadrilateral Properties

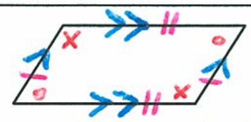
→ means parallel lines

Trapezoid



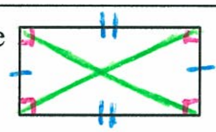
One set of parallel lines.

Parallelogram



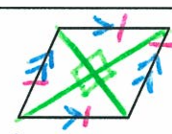
- 2 sets of equal parallel lines
- Opposite Angles are equal
- Adjacent angles (next to each other) are supplementary.

Rectangle



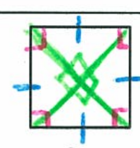
- All angles 90°
- 2 sets of equal/parallel lines
- Diagonals are equal
- Diagonals bisect each other (cut in half at intersection)

Rhombus



- 2 sets of parallel lines
- All 4 sides equal length.
- Diagonals bisect at 90° (cut in half & meet at 90°)

Square



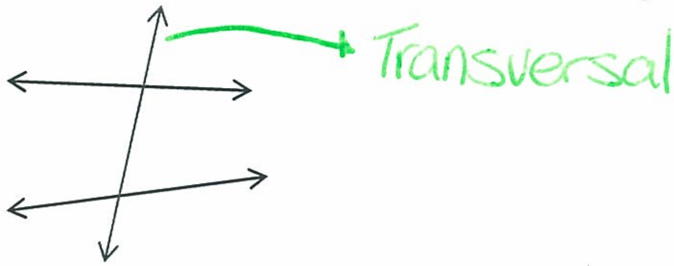
- opposite sides parallel
- All sides equal
- 4 right angles
- diagonals bisect at 90° (cut in half & meet at 90°)

Assignment: Geometric Properties Worksheet

2.1 Exploring Parallel Lines

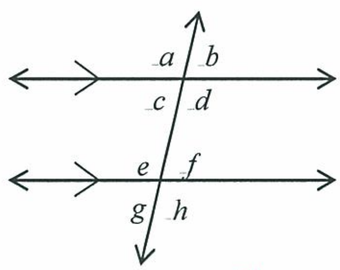
Parallel Lines and Transversals

A **transversal** is a line that intersects two or more other lines at distinct points.



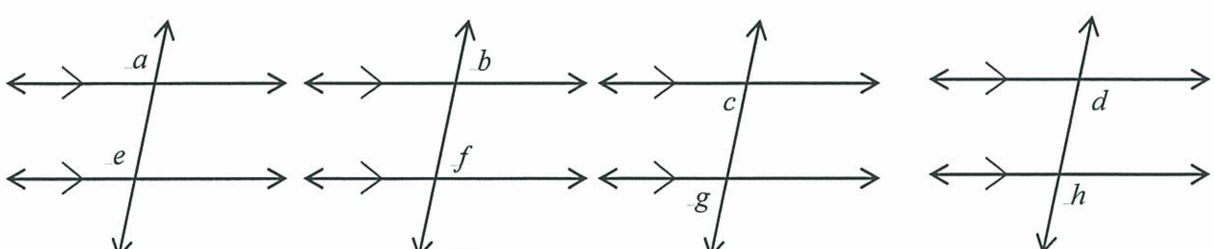
Parallel lines are lines with the same slope but different *y*-intercepts. Parallel lines will never intersect each other.

If two parallel lines are cut by a transversal, eight angles are created.



In the same position.

Corresponding angles are on the same side of the transversal, and on the same side of the parallel lines. (They are in the same position)

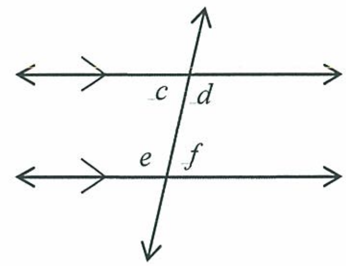


a = e   b = f   c = g   d = h

Corresponding Angles are <u>Equal</u>

Interior angles lie inside the parallel lines.

Co-Interior Angles: Interior angles on the same side of the transversal.



Co-Interior Angles are Supplementary

$c+e=180°$ & $d+f=180°$

Alternate Interior Angles: Interior Angles on opposite sides of the transversal.

Alternate Interior Angles are Equal

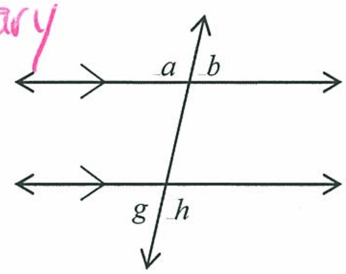
$c=f$ & $d=e$

Exterior angles lie outside the parallel lines.

Co-Exterior Angles: Exterior angles on the same side of the transversal.

Co-Exterior Angles are Supplementary

$a+g=180°$ $b+h=180°$



Alternate Exterior Angles: Exterior angles on opposite sides of the transversal.

Alternate Exterior Angles are equal

$a=h$ $b=g$

If two parallel lines are cut by a transversal then Corresponding Angles, Alternate Interior Angles, & Alternate Exterior Angles are equal.

(used to prove (or find) angle measures)

Likewise, if two lines are cut by a transversal and the Corresponding Angles, or Alternate Interior Angles, or the Alternate Exterior Angles are equal then the lines are parallel.

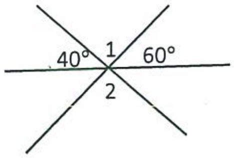
(used to prove lines are parallel)

~~A~~-must give reasons!

Example 1: Find each indicated angle:

"Angle 1"

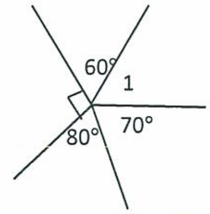
a.



$\angle 1 = 80°$ Angles on Line

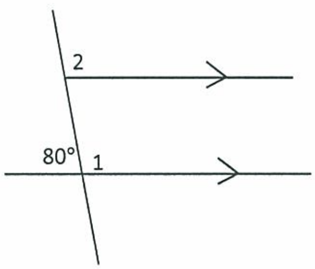
$\angle 2 = 80°$ Vertically Opposite

b.



$\angle 1 = 60$ Angles at a Point.

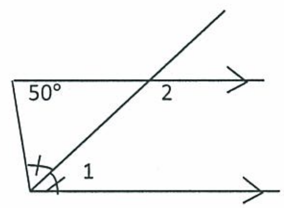
c.



$\angle 1 = 100°$ Supplementary Angles

$\angle 2 = 100°$ Corresponding Angles

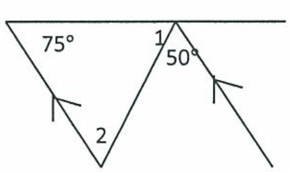
d.



$\angle 1 = 65°$ Half of Co-Interior Angle (50°)

$\angle 2 = 115$ Co-Interior Angles

e.



$\angle 1 = 55°$

Co-Interior with 70° & 50°

$\angle 2 = 50°$ $\angle$ Sum $\triangle$ (Angle sum of triangle)

(or Alt-Interior)

Assignment: Pg. 72 #2-6

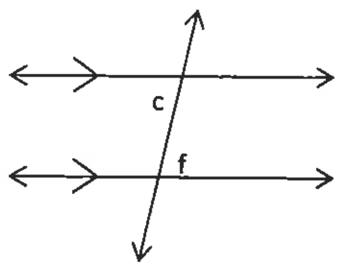
OR $\angle 2 = 50°$ Alt-Interior

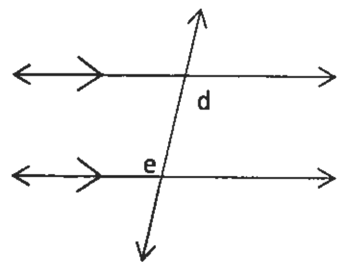
$\angle 1 = 55°$ Angle Sum of Triangle.

FOM 11 **2.2 Angles Formed by Parallel Lines**

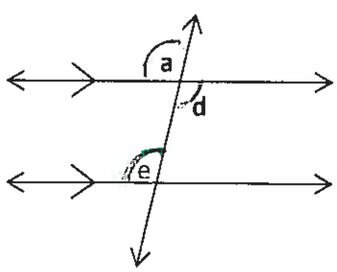
From last day we know that when a transversal crosses parallel lines, the corresponding angles are equal. There are two other sets of angles that have a relationship when a transversal crosses parallel lines.

Alternate Interior Angles
When a transversal intersects a pair of parallel lines, the **alternate interior angles** are equal.





Proof:



$\angle a = \angle e$ Corr $\angle$'s $=$

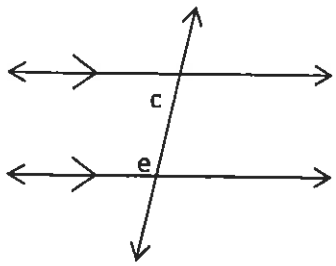
$\angle a = \angle d$ Vert Opp $\angle$'s $=$

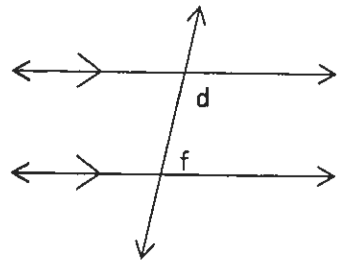
$\angle d = \angle e$ both $= \angle a$

→ add to 180°

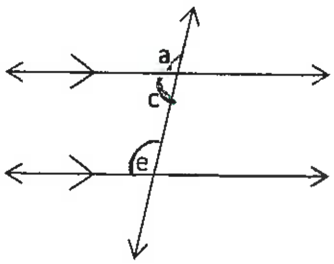
Co-Interior Angles:

When a transversal intersects a pair of parallel lines, the **co-interior angles** are supplementary.





Proof:



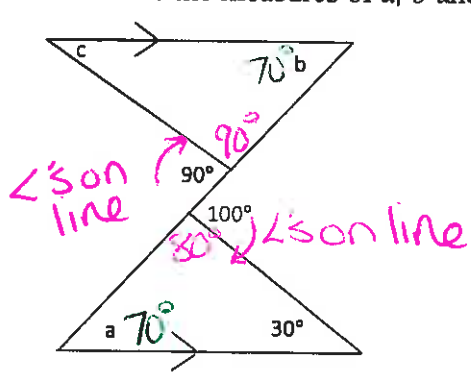
$\angle a = \angle e$ → Corr $\angle$'s =

$\angle a + \angle c = 180°$ → $\angle$'s on line = 180°

$\angle e + \angle c = 180°$ → because $\angle e = \angle a$ (substitution)

∴ $\angle e + \angle c$ are supplementary (2 angles that add to 180°)

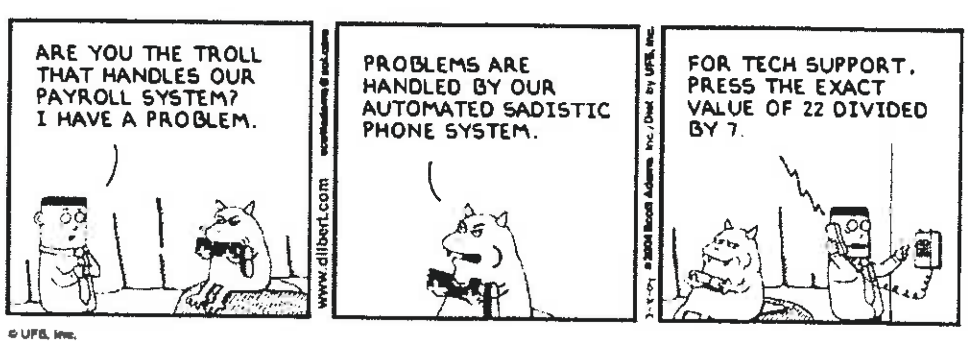
Example 1: Determine the measures of *a*, *b* and *c*.



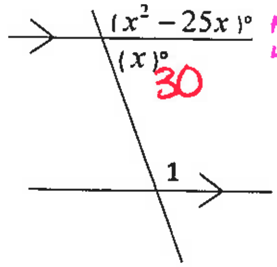
$\angle a = 70°$ → $\angle$ sum △

$\angle b = 70°$ → Alt. Int $\angle$'s =

$\angle c = 20°$ → $\angle$ sum △



Example 2: Find the measure of $\angle 1$.



must add to 180°

$$x^2-25x+x=180^\circ$$
$$x^2-24x=180^\circ$$
$$x^2-24x-180^\circ=0$$

$\underline{6} \times \underline{-30}$ -180
$\underline{6} + \underline{-30}$ -24

	x	$+6$
x	x^2	$6x$
-30	$-30x$	-180

$$(x-30)(x+6)=0$$

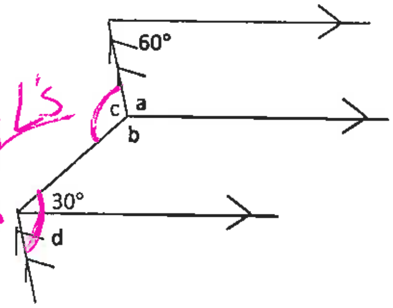
$\therefore$ $x=30$ OR -6

Reject (can't have negative angle)

$\Rightarrow$ $\angle 1 = 150^\circ$ - Co-Interior Angles

Example 3: Determine the measures of *a, b, c* and *d.*

Alt-Int $\angle$'s



$\angle a = 120^\circ$ Co-Interior $\angle$'s

$\angle b = 150^\circ$ Co-Interior $\angle$'s

$\angle c = 90^\circ$ $\angle$'s at point $= 360^\circ$

$\angle d = 60^\circ$ Alt-Int $\angle$'s

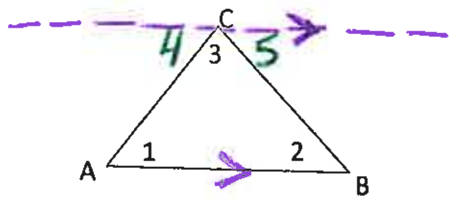
Assignment: pg. 78 #1-4, 10, 12, 13, 15, 16, 20

FOM 11 **2.3 Angle Properties In Triangles**

The sum of the angles in a triangle is 180°.

We can use our knowledge of parallel lines to prove (deductively) this theorem.

Example 1: Given ΔABC, prove $\angle 1 + \angle 2 + \angle 3 = 180°$.



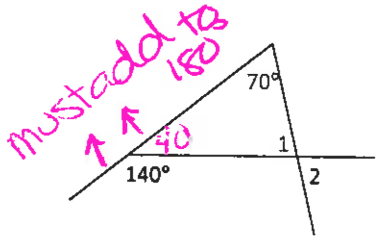
$\angle 1 = \angle 4$ Alt Int $\angle$'s

$\angle 2 = \angle 5$ Alt Int $\angle$'s

$\angle 3 + \angle 4 + \angle 5 = 180°$ $\angle$'s on line $= 180°$

$\boxed{\angle 3 + \angle 1 + \angle 2 = 180°}$ Substitution

Example 2: Determine the measures of $\angle 1$ and $\angle 2$.

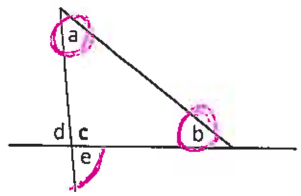


$\angle 1 = 70°$ $\angle$ Sum Δ

$\angle 2 = 70°$ Vert Opp $\angle$'s

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles.

Example 3: Prove $\angle e = \angle a + \angle b$.

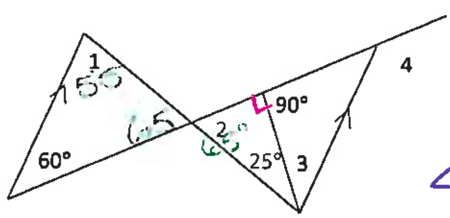


$\angle e + \angle c = 180°$ $\angle$'s on line

$\angle a + \angle b + \angle c = 180°$ $\angle$ Sum $\triangle$

$\angle e = \angle a + \angle b$ (both add to 180° with $\angle c$)

Example 4: Determine $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$.



$\angle 2 = 65°$ $\angle$ Sum $\triangle$

$\angle 1 = 55°$ $\angle$ Sum $\triangle$

$\angle 3 + 25° = 55$ Alt-Int $\angle$'s

$\therefore \angle 3 = 30°$

$\angle 4 = 90 + 30$ (ext. Angle = sum of non-adjacent interior)

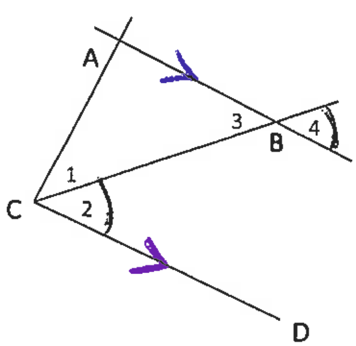
$\angle 4 = 120°$

Example 5: Given $AB \parallel CD$

&

$\angle 1 = \angle 4$

Prove $\angle 1 = \angle 2$



Statement	Reason
$AB \parallel CD$	Given
$\angle 2 = \angle 4$	Corresponding $\angle$'s
$\angle 1 = \angle 4$	Given
$\angle 2 = \angle 1$	both = $\angle 4$

Assignment: pg. 90 #2, 3, 5-9, 12, 15, 16, 18

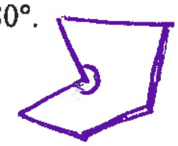
FOM 11

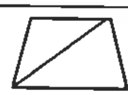
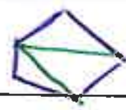
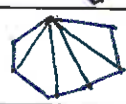
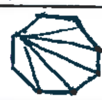
2.4 Angle Properties in Polygons

A **polygon** is a closed geometric figure made up of *n* straight sides.

A **convex polygon** has all interior angles less than 180°.

A **concave polygon** has at least one interior angle greater than 180°.



# of sides in a polygon	sketch	# of triangles formed	Sum of interior angles of the polygon
3 TRIANGLE		1	$1 \times 180° = 180°$
4 QUADRILATERAL		2	$2 \times 180° = 360°$
5 PENTAGON		3	$3 \times 180 = 540°$
6 HEXAGON		4	$4 \times 180° = 720°$
7 HEPTAGON		5	$5 \times 180 = 900°$
8 OCTAGON		6	$6 \times 180 = 1080°$
9 NONAGON		7	$7 \times 180 = 1260°$
10 DECAGON	etc	8	$8 \times 180 = 1440°$
11 "HENDECAGON"		9	$9 \times 180 = 1620°$
12 DODECAGON		10	$10 \times 180 = 1800°$
n POLYGON		n-2	$(n-2)180 = 180(n-2)$

All sides are equal & all angles

In any polygon with n sides, the sum of the interior angles is $180°(n-2)$. A **regular polygon** has equal sides and equal angles.

(15 triangles)

Example 1: Determine the measure of each interior angle of a regular 17-sided polygon.

Sum of interior angles $= 180(n-2)$

$= 180(17-2)$

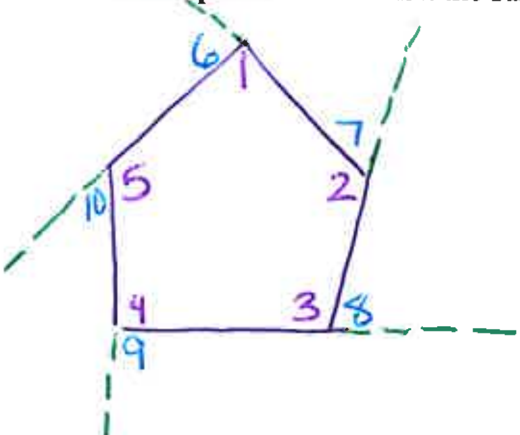
$= 180(15) = 2700°$

$\therefore$ Each Int $\angle$ must be:

$\frac{2700}{17} = \boxed{158.82°}$

The sum of the exterior angles of any convex polygon is 360°.
Each exterior angle of a regular polygon is $\frac{360^o}{n}$.

Example 2: Show that the sum of the exterior angles of a pentagon is 360°.



$\angle 1 + \angle 6 = 180°$

$\angle 2 + \angle 7 = 180°$

$\angle 3 + \angle 8 = 180°$

$\angle 4 + \angle 9 = 180°$

$\angle 5 + \angle 10 = 180°$

$\angle 1 + \angle 2 + \ldots + \angle 9 + \angle 10 = 5 \times 180°$

$\Rightarrow 5 \times 180° =$ Sum of all int & ext $\angle$'s

(Sum int $\angle$'s $= 3(180)°$)

$5 \times 180 = 3(180) + \text{ext} \angle\text{'s}$

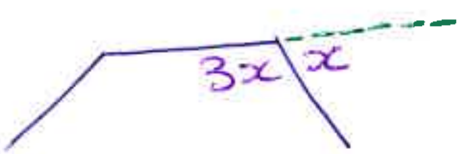
$900° = 540 + \text{ext} \angle\text{'s}$

$-540 \quad -540$

$\boxed{360° = \text{Sum Ext} \angle\text{'s}}$

Example 3: What type of regular polygon has an interior angle 3 times the exterior angle?

(Int = 3 × Ext)



$x + 3x = 180°$

$\frac{4x}{4} = \frac{180°}{4}$

$x = 45°$

Rule From Above:

Ext $\angle = \frac{360}{n}$

$n \times 45 = \frac{360}{\cancel{n}} \times \cancel{n}$

$\frac{45n}{45} = \frac{360}{45}$

$n = 8$

Assignment: Pg. 99 #1-4, 6-11, 14, 18

$\therefore$ An Octagon has interior angles that are 3 times the exterior $\angle$'s